MATH 251
Examination II
November 4, 2008

Name: ____________

Student Number: ____________

Section: ____________

This exam has 13 questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM.
PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE.

Do not write in this box.

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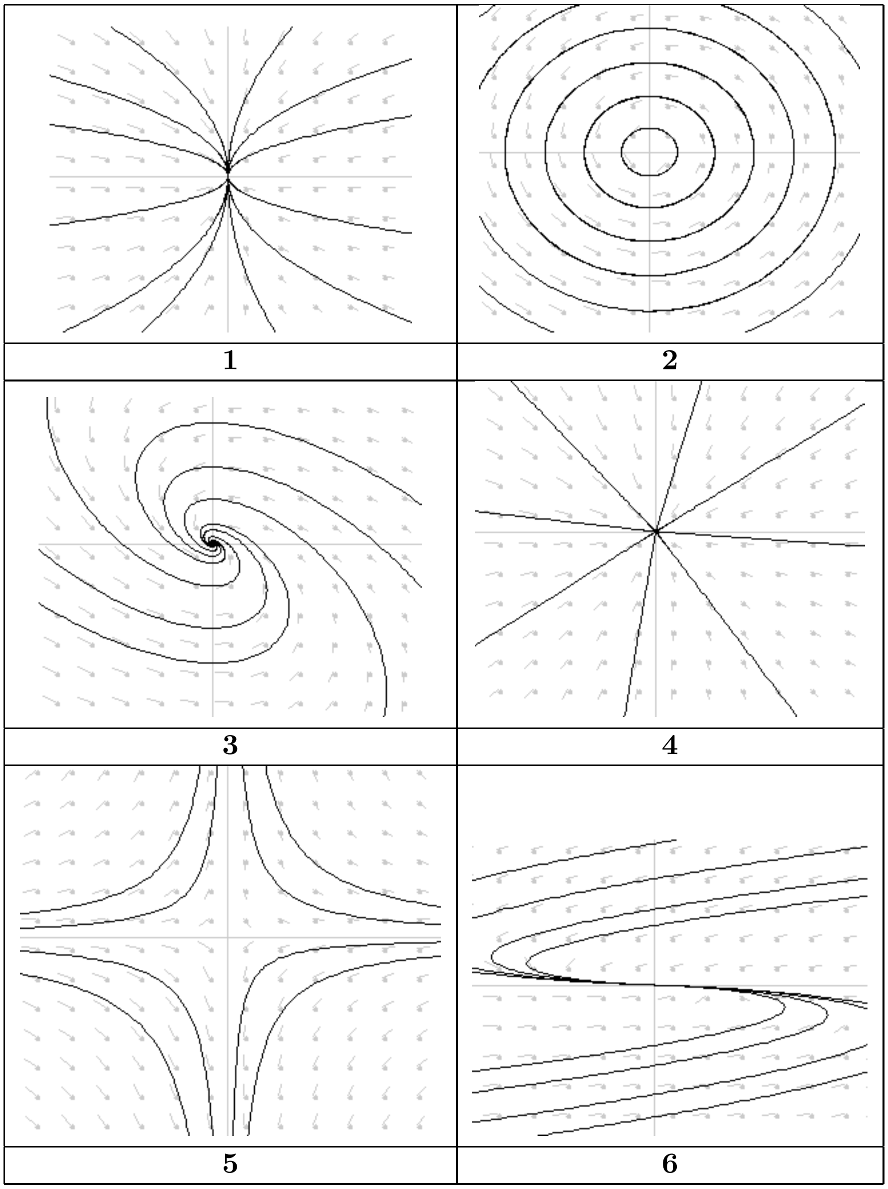
1. (5 points) Match the sketches of phase portraits for 2x2 homogeneous linear systems $\mathbf{x}' = A\mathbf{x}$ with the names of their critical points at the origin.



(a) saddle ________
(b) node ________
(c) proper node ________
(d) center ________
(e) spiral ________

2. (5 points) Match the following formulas for general solutions of 2x2 homogeneous linear systems $\mathbf{x}' = A\mathbf{x}$ with the sketches of the phase portraits given in Problem 1:

(a) $c_1 e^{2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ______

(b) $c_1 e^{t}\begin{pmatrix} \cos t \\ 2\sin t \end{pmatrix} + c_2 e^{t}\begin{pmatrix} \sin t \\ 2\cos t \end{pmatrix}$ ______

(c) $c_1 e^{-t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ______

(d) $c_1 e^{t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ______

(e) $c_1 \begin{pmatrix} \cos t \\ 2\sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ 2\cos t \end{pmatrix}$ ______

3. (5 points) Match the three adjectives for the critical point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ of a 2×2 homogeneous linear systems $\mathbf{x}' = A\mathbf{x}$ with the five general solutions given in the table below by placing one of the letters **A**, **U**, or **S** in each of the five blanks.

(a) $c_1 e^{2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ______

(b) $c_1 e^{t}\begin{pmatrix} \cos t \\ 2\sin t \end{pmatrix} + c_2 e^{t}\begin{pmatrix} \sin t \\ 2\cos t \end{pmatrix}$ ______

(c) $c_1 e^{-t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ______

(d) $c_1 e^{t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ______

(e) $c_1 \begin{pmatrix} \cos t \\ 2\sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ 2\cos t \end{pmatrix}$ ______

Use

A for asymptotically stable

U for unstable

S for stable

4. (5 points) When an object with mass 5 kg is attached to a spring, the object stretches the spring by 2 m. A damper with damping coefficient of 4 N-s/m is attached to the system. Assume there is no external force acting on the system and that acceleration due to gravity is 10 m/s^2. If the object is released 1 m above its equilibrium position and is given an initial downward velocity of 3 m/s. Assuming that the downward direction is positive, which initial value problem describes the motion of the system?

(a) $5y'' + 4y' + 25y = 0, \quad y(0) = 1, \quad y'(0) = -3$
(b) $5y'' + 4y' + 25y = 0, \quad y(0) = -1, \quad y'(0) = 3$
(c) $5y'' + 4y' + 10y = 0, \quad y(0) = 1, \quad y'(0) = -3$
(d) $5y'' + 4y' + 10y = 0, \quad y(0) = -1, \quad y'(0) = 3$

5. (5 points) Initially an object with mass 2 kg is located on a frictionless surface and is moving eastward with no forces acting on it. At time $t = 3$ seconds a constant external eastward force of .17 Newtons is added. At $t = 4$ seconds the object is struck with a hammer in such a fashion that its momentum is reduced by .34 kg-m/s at that time. Which of the following $F(t)$ represents the combination of these two forces.

(a) $F(t) = .17u(t-3) + .34\delta(t-4)$
(b) $F(t) = .17\delta(t-3) + .34u(t-4)$
(c) $F(t) = .17u(t-3) - .34\delta(t-4)$
(d) $F(t) = -.17\delta(t-3) + .34u(t-4)$

6. (5 points) What is the general real-valued solution of the following ODE:

$$y''' + y' = 0$$

(a) $y(t) = c_1 e^t + c_2 e^{-t} + c_2$
(b) $y(t) = c_1 \cos t + c_2 \sin t + c_2 e^t$
(c) $y(t) = c_1 + c_2 \cos t + c_3 \sin t$
(d) $y(t) = c_1 t^2 + c_2 t + c_3$

7. (5 points) A spring-mass system is described by the differential equation $y'' + 2y' + 10y = 0$ with initial conditions $y(0) = 1$ and $y'(0) = -1$ Which statement will best describe the behavior of the solution $y(t)$ after a long time?

   (a) $y(t)$ will oscillate with increasing amplitude.
   (b) $\lim_{t\to\infty} y(t) = 0$
   (c) $y(t)$ will approach a periodic function with period $\frac{2\pi}{3}$.
   (d) $y(t)$ will approach a periodic function with period π.

8. (5 points) $\int_0^\infty e^{-(s-2)t}\, dt$ is the Laplace transform of which of the following functions?

   (a) t^2
   (b) e^{2t}
   (c) $u(t-2)$
   (d) $\delta(t-2)$

9. (5 points) If $f(t) = u(t-2) - 2u(t-3) + 3u(t-4) - 4u(t-5)$, what is $f(\pi)$?

   (a) -2
   (b) -1
   (c) 0
   (d) 1

10. A spring-mass system is modeled by the initial value problem

$$2y'' + \gamma y' + 18y = F(t), \quad y(0) = 3, \quad y'(0) = -12.$$

(a) (6 points) If $\gamma = 0$ and $F(t) = 0$, what is the amplitude of displacement?

(b) (3 points) If $\gamma = 0$ and $F(t) = 2\cos(\omega t)$, for which value(s) of ω will the system undergo resonance?

(c) (3 points) If $F(t) = 0$, for which value(s) of γ will the system ***not*** oscillate?

11. Compute the following:

(a) (7 points)

$$\mathcal{L}\{e^{3t}u(t-2)\}.$$

(b) (7 points)

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+8s+25}\right\}.$$

12. (16 points) Use Laplace transforms to solve the following initial value problem:

$$y' + 3y = tu(t-2) + \delta(t-3), \quad y(0) = 1.$$

13. Consider the 2×2 linear homogeneous system:

$$\mathbf{x}' = \begin{pmatrix} 0 & 4 \\ -1 & 0 \end{pmatrix} \mathbf{x}.$$

(a) (2 points) Find the vector $\mathbf{x}'$ at the point $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

(b) (11 points) Find the solution $\mathbf{x}(t)$ that satisfies the initial condition $\mathbf{x}(0) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$.