

MATH 251
Fall 2003
Exam 2
November 10, 2003

NAME : _____

ID : _____

INSTRUCTOR : _____

There are **10** questions on **9** pages. Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice. For partial credit questions, all work must be shown — **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.
PLEASE DO NOT WRITE IN THE BOX BELOW.

1: _____
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9: _____
10: _____
Total: _____

1. (5 points) A spring-mass system, subject to an external force of $10 \cos(2t)$ Newtons, is equipped with a spring with a Hooke's constant 12 Newtons per meter. For what mass will the resonance occur?
- (a) 2 kg
 - (b) 3 kg
 - (c) 6 kg
 - (d) 10 kg

2. (5 points) Of what form will the particular solution to the following differential equation be? Do not solve the equation.

$$y'' - 4y' + 4y = e^{2t} + t^2 e^{3t} - \sin(2\pi t)$$

- (a) $Ae^{2t} + Bt^2 e^{3t} + Cte^{3t} + De^{3t} + E \sin(2\pi t) + F \cos(2\pi t)$
- (b) $Ate^{2t} + Bt^2 e^{3t} + Cte^{3t} + De^{3t} + E \sin(2\pi t) + F \cos(2\pi t)$
- (c) $At^2 e^{2t} + Bt^2 e^{3t} + Cte^{3t} + De^{3t} + E \sin(2\pi t) + F \cos(2\pi t)$
- (d) $Ae^{2t} + Bt^2 e^{3t} + E \sin(2\pi t) + F \cos(2\pi t)$

3. (5 points) The second order linear equation $y'' - 4y' + 4y = 0$ is equivalent to

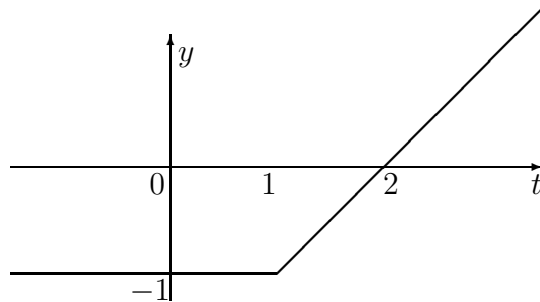
(a)
$$\begin{cases} x'_1 = x_2 \\ x'_2 = -4x_1 + 4x_2 \end{cases}$$

(b)
$$\begin{cases} x'_1 = x_1 \\ x'_2 = -4x_1 + 4x_2 \end{cases}$$

(c)
$$\begin{cases} x'_1 = x_2 \\ x'_2 = 4x_1 - 4x_2 \end{cases}$$

(d)
$$\begin{cases} x'_1 = x_1 \\ x'_2 = 4x_1 - 4x_2 \end{cases}$$

4. (5 points) Which of the following functions corresponds to this graph:



(a) $tu_1(t) - 1$

(b) $tu_1(t - 1) - 1$

(c) $(t - 1)u_1(t) - 1$

(d) $(t - 1)u_1(t - 1) - 1$

5. (14 points) Solve the following initial value problem:

$$y'' - 5y' - 14y = -14t^2 - 10t - 26, \quad y(0) = 0, \quad y'(0) = 13$$

6. (14 points) Rewrite the following function in terms of step functions and find its Laplace transform.

$$f(t) = \begin{cases} 1, & t < 1 \\ 2t - 1, & 1 \leq t < 2 \\ 3, & t \geq 2 \end{cases}$$

7. (10 points) Assume that $g = 10 \frac{\text{m}}{\text{sec}^2}$ for this problem.
- (a) Mass of 2 kg is weighting down a spring by 80 cm. Calculate the spring constant.
- (b) If the above mentioned spring–mass system is placed into the liquid with damping constant 14 Newton-seconds per meter, and no external force is in effect, what would be the quasi–period of the vibration?

8. (12 points) Calculate the inverse Laplace transform of

$$\frac{s^2 + 9}{9s - s^3}$$

9. (20 points) Solve the following initial value problem:

$$y'' + 2y' + 10y = 10 + \delta(t), \quad y(0) = 1, \quad y'(0) = 0$$

10. (10 points) Solve the initial value problem

$$X' = \begin{bmatrix} 3 & 3 \\ 4 & 2 \end{bmatrix} X, \quad X(0) = \begin{bmatrix} -8 \\ 13 \end{bmatrix}$$