

MATH 251
2nd exam
Nov. 12, 2002

Name: _____
Student Number: _____
Instructor: _____
Section: _____

There are 9 partial credit questions. **In order to obtain full credit for these problems, all work must be shown. Credit will not be given for an answer not supported by work.**

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION. At the end of the examination, the booklet will be collected.

1. Transform the given initial value problem into an initial value problem for two first order equations.

$$x^2y'' + xy' + (x^2 - \lambda^2)y = 0, \quad y(0) = y_0, \quad y'(0) = y'_0$$

2. What is the form of the particular solution to the equation

$$y'' + 4y = e^{2t} + \sin(2t) - 3\cos(t) + t^2 - 4$$

DO NOT SOLVE FOR THE CONSTANTS!

3. (a) Sketch the function.

$$f(t) = t - u_1(t)(t - 1) + u_3(t)(2 - t)$$

- (b) Write the given function in terms of unit step functions.

$$g(t) = \begin{cases} t^2 & 0 \leq t < 2 \\ 6 - t & 2 \leq t < 6 \\ 0 & 6 \leq t \end{cases}$$

4. A mass of 3 kg is attached to a spring with spring constant $k = 12 \frac{\text{N}}{\text{m}}$. What value of the damping coefficient γ will make the system critically damped?

5. Find the Laplace transform.

$$\mathcal{L}\{(e^{\pi t} \sin(3t))'''\}$$

$$\text{HINT:} \left(\begin{array}{l} (e^{\pi t} \sin(3t))' = \pi e^{\pi t} \sin(3t) + 3e^{\pi t} \cos(3t) \\ (e^{\pi t} \sin(3t))'' = (\pi^2 + 9)e^{\pi t} \sin(3t) + 6\pi e^{\pi t} \cos(3t) \end{array} \right)$$

6. Find the solution to the following initial value problem.

$$y'' + 2y' - 8y = e^{3t}u_3(t), \quad y(0) = 0, \quad y'(0) = 1$$

7. Solve the initial value problem.

$$y'' + 5y' + 4y = 2\delta(t - 3), \quad y(0) = 1, \quad y'(0) = 0$$

8. Find the general solution. $\vec{x}' = \begin{pmatrix} 1 & 4 \\ 3 & 0 \end{pmatrix} \vec{x}$, $\vec{x} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$.

9. Find the general solution to the following systems of equations.

$$(a) \vec{x}' = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix} \vec{x}$$

$$(b) \vec{x}' = \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix} \vec{x}$$