MATH 251
2nd exam
Oct. 31, 2001

Name:_____
Student Number:_____
Instructor:_____
Section:_____

There are **7** partial credit questions. **In order to obtain full credit for these problems, all work must be shown. Credit will not be given for an answer not supported by work.**
THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION. At the end of the examination, the booklet will be collected.

1:_____
2:_____
3:_____
4:_____
5:_____
6:_____
7:_____
Total:_____

Do not write in the box to the left

1. (15 pts)

   Solve the initial value problem.

$$y'' + 4y = e^t, \qquad y(0) = 0, \qquad y'(0) = 0$$

2. (15 pts)

   What is the form of the general solution to the following ordinary differential equation? **Do <u>not</u> solve for the constants!**

$$y'' - y = \cos(2t) + 3te^t - 4\sin(t)$$

3. (a) (3 pts) Circle the correct answer. According to the definition,

$$\mathcal{L}\{f(t)\} =$$

1. $\int_0^\infty e^{st} f(t)\, dt$
2. $\int_0^\infty e^{-st} f(t-c)\, dt$
3. $\int_0^\infty e^{-st} f(t)\, dt$

(b) (3 pts) Circle the correct term.

Suppose A is a 2×2 matrix with repeated eigenvalue λ. Is $\vec{x} = c_1 e^{\lambda t}\vec{v_\lambda} + c_2 t e^{\lambda t}\vec{v_\lambda}$ the general solution to $\vec{x}' = A\vec{x}$, where $\vec{v_\lambda}$ is an eigenvector for λ?

1. Yes
2. No

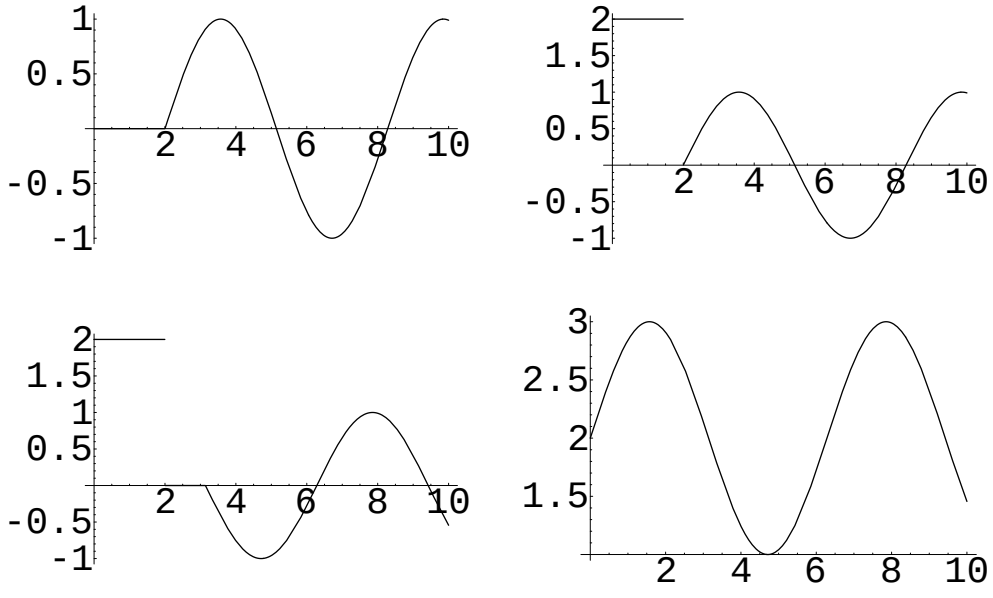
(c) (3 pts) True or False

$$\mathcal{L}\{f(t)g(t)\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$$

(d) (3 pts)

Circle the graph of the following function.

$$2(1 - u_2(t)) + \sin(t)u_\pi(t)$$



4. (15 pts) Rewrite the following function in terms of unit step functions and find its Laplace transform.

$$f(x) = \begin{cases} t^2 & 0 \leq t < 2 \\ 1 & 2 \leq t \end{cases}$$

5. A 2 kg mass is hung on a damped spring of spring constant $20\,\frac{\text{kg}}{\text{s}^2}$ with linear damping factor of $4\,\frac{\text{kg}}{\text{s}}$, and set into motion.

   (a) (10 pts)
   Set up the equation that models the motion of the mass **and** find the genaral solution.

   (b) (3 pts) At what (quasi) frequency does the system oscillate?

   (c) (2 pts) How many times will the mass pass through its equilibrium position?

6. (a) (10 pts)
Solve the initial value problem

$$y'' + 8y' + 16y = \delta(t-1), \qquad y(0) = 0, \qquad y'(0) = 0$$

(b) (4 pts)
At what time(s) does the solution attain its maximum?

7. (a) (10 pts)

Solve the initial value problem. $\vec{x'} = \begin{pmatrix} -2 & 3 \\ 2 & 3 \end{pmatrix} \vec{x}; \qquad \vec{x}(0) = \begin{pmatrix} 6 \\ \alpha \end{pmatrix}$

(b) (4 pts)

For what value α is $\lim_{t \to \infty} \vec{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$?