BE SURE TO WRITE YOUR NAME, SECTION, AND SOCIAL SECURITY NUMBER ON THE FRONT PAGE.

MATH 251 MIDTERM EXAMINATION II November 1, 2000

The exam consists of 6 problems. The work involved in solving the problems of the exam has to be fully shown in order to get partial credit. No points will be assigned to problems with an unjustified answer, even if correct. Please, show all your work clearly and neatly. The point value for each question is in parantheses to the right of the question number.

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE ?? PROBLEMS ON ?? PAGES (INCLUDING THIS ONE).

1. (16 pts.) The function $y_1(t) = t^3$ is a solution of the differential equation

$$t^2y'' - 6y = 0,\ t > 0.$$

Find a second solution of this differential equation.

2. (20 pts.) a) Find the general solution of the differential equation

$$y'' - 6y' + 9y = 3e^{2t}.$$

b) What is the form of the particular solution of the differential equation

$$y'' - 6y' + 9y = 4t \sin 2t + t^3 + e^{3t}.$$

Do not solve for the constants in part (b)!

3. (16 pts.) Assume that $y_1(t) = t$ and $y_2(t) = te^t$ are solutions of the differential equation

$$t^2y'' - t(t+2)y' + (t+2)y = 0, \ t > 0.$$

(a) Do $y_1(t)$ and $y_2(t)$ form a fundamental set of solutions of the O.D.E.? **Justify your answer.**

(b) State the general solution for this O.D.E..

4. (16 pts.) A mass of 2 kg stretches a spring 1 m. The mass is in a medium that exerts a viscous resistance of 8 newtons when the velocity of the mass is 2 m/sec. The mass is stretched from its equilibrium position 1 m and set in motion with a downward velocity of $(3\sqrt{3}-1)$ m/sec. Let $g = 10$ m/sec^2.

   a) Set up and solve the initial value problem for $u(t)$, the displacement of the mass from its equilibrium position.

   b) What is the limit of the solution as $t \to \infty$? **Justify your answer.**

   c) How many times does the mass pass through its equilibrium position? **Justify your answer.**

5. (16 pts.) a) Find a function y(t) whose Laplace transform is the expression $\frac{s+4}{s^2-4s+13}$.

(b) Rewrite the following function f(t) in terms of step functions, and find its Laplace transform.

$$f(t) = \begin{cases} 3t^2, & 0 \leq t < 2\pi \\ 3t^2 + \sin \text{ t}, & t \geq 2\pi. \end{cases}$$

6. (16 pts.) Solve the initial value problem, $y'' - 4y = \delta(t - 2\pi)$, $y(0) = 0$, $y'(0) = 1$.