This exam has 9 questions for a total of 100 points. In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.
At the end of the examination, the booklet will be collected.
1. (10 points) Use the Laplace transform to solve the equation \( y' + 2y = 2e^{4t} \), with initial condition \( y(0) = -1 \). You must use the Laplace transform to receive any credit.
2. (10 points) Solve the initial value problem

\[ y'' + 4y = \delta(t - 1), \quad y(0) = 1, \quad y'(0) = 0. \]
3. (12 points) Express the following piecewise continuous function in terms of step functions and then find its Laplace transform:

\[ f(t) = \begin{cases} 
2t & 0 \leq t < 2 \\
2^2 - t & 2 \leq t < 2 \\
0 & t \geq 3.
\end{cases} \]
4. (15 points) Find the Laplace transform of the following functions.

a) \( e^{-3t} \cos 5t \)

b) \( te^{5t} \)

c) \( u_{\pi/2}(t) \sin t \)

Hint: Recall that \( \sin(t + \pi/2) = \cos(t) \).
5. (10 points) Find the inverse Laplace transform of
a) \[ \frac{3s + 4}{s^2 + 4s + 20} \]
b) \[ \frac{e^{-8s}}{(s - 9)^2} \]
6. (5 points) Suppose the phase portrait of the system

\[ x' = \begin{bmatrix} 2 & 1 \\ -1 & d \end{bmatrix} x \]

has an improper node at \((0, 0)\). Find the value (or range of values) of \(d\).
7. (5 points) Find the general solution of the system

\[ x' = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} x. \]
8. (20 points) Consider the system of linear equations

\[ x' = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} x; \quad x(0) = \begin{bmatrix} 1 \\ -4 \end{bmatrix}. \]

Solve the initial value problem and classify the type and stability of the critical point at \((0,0)\).
9. (13 points) The displacement $y(t)$ of a mass-spring system is described by the equation

$$y'' + 4y' + 3y = g(t),$$

where

$$g(t) = \begin{cases} 
1, & 0 \leq t < 10 \\
0, & t \geq 10. 
\end{cases}$$

At $t = 0$, the system is set in motion from equilibrium position with no initial velocity.

a) Find the displacement function $y(t)$.

b) What is the displacement when $t = 5$?