

MATH 251
Examination I
June 29, 2015
FORM A

Name: _____
Student Number: _____
Section: _____

This exam has 11 questions for a total of 100 points. Show all your work! **In order to obtain full credit for partial credit problems, all work must be shown. Points might be deducted, at the sole discretion of the instructor, for an answer not supported by a reasonable amount of work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM. PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE.

Do not write in this box.

1-2: _____
3: _____
4: _____
5: _____
6-7: _____
8: _____
9: _____
10: _____
11: _____
Total: _____

1. (7 points) For each problems below, determine the order of the given differential equation; also state whether the equation is linear or nonlinear. **If the equation is nonlinear, please circle the term(s) that make it so.**

Differential equations	Order	Linear/Nonlinear
$y'' + yy' - 2ty = 0$ $y''' - e^{-5t}y' + (\sin t)y = 4t^2 - 7$ $2t^2y^{(4)} + ty' - 6y = (12 - t - t^2)e^{\frac{t}{2}}$ $\frac{dy}{dt} + 3y = 6 - \tan 2t$ $\left(\frac{d^3y}{dt^3}\right)^2 - \left(\frac{d^2y}{dt^2}\right)^3 + \frac{dy}{dt} = 0$		

2. (6 points) Give an example of the following:

(a) (2 points) A third order partial differential equation.

(b) (2 points) A third order, linear, homogeneous, ordinary differential equation.

(c) (2 points) A first order, autonomous, ordinary differential equation.

3. (10 points) Solve **explicitly** the initial value problem;

$$y' + 9xe^{y+3x} = 0, \quad y(0) = 0.$$

4. (10 points) Find all real number α such that the particular solution $y = \phi(t)$ of the following initial value problem remains finite as $t \rightarrow \infty$. Justify your answer.

$$y' - y = \frac{1 - 2t}{e^t}, \quad y(0) = \alpha.$$

5. (10 points) Solve the initial value problem;

$$(2x \sin y - y \cos x) + (x^2 \cos y - \sin x)y' = 0, \quad y\left(\frac{\pi}{2}\right) = 0.$$

You may leave your answer in implicit form.

6. (6 points) Find the largest interval on which the solution of

$$(2 - \ln t)y' + 3y = 4t^2, \quad y(2) = 10$$

is guaranteed to exist, without solving the initial value problem itself. (Hint: e is approximately 2.718)

7. (6 points) A 600-gallon tank is initially filled with 400 gallons of water with a salt concentration of 1 pound per gallon. A salt water mixture with a concentration of 3 pounds per gallon enters the tank at the rate of 20 gallons per minute. Then a thoroughly mixed solution leaves the tank at the rate of 24 gallons per minute. Let $Q(t)$ be the quantity of salt in the tank at time t . (pounds) Find the initial value problem which accurately describes the situation.

8. (13 points) Consider the following differential equation

$$y' = y(1 + y)^2(2 - y)^3.$$

- (a) (3 points) Find all of its equilibrium solutions.

- (b) (4 points) Classify the stability of each equilibrium solution. Justify your answer.

- (c) (2 points) If $y(-3) = -1$, what is $y(-9)$? Without solving the equation, briefly explain your conclusion.

- (d) (4 points) Suppose $y(3) = \alpha$ and $\lim_{t \rightarrow \infty} y(t) = 2$. Find the value(s) of α .

9. (10 points) Find the particular solutions of the following initial value problem. Express your answer in terms of real valued functions.

$$y'' + 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = -5.$$

Find also $\lim_{t \rightarrow \infty} y(t)$.

10. (12 points) Given that $y_1(t) = t^{-1}$ is a solution to the equation,

$$t^2 y'' + 3ty' + y = 0, \quad t > 0.$$

Use the method of reduction of order to find another solution y_2 .

11. (10 points) Consider the second order linear differential equation

$$t^2 y'' + 2ty' + 3y = 0.$$

Suppose $y_1(t)$ and $y_2(t)$ are two fundamental sets of solutions of the equation satisfying

$$y_1(1) = 4, \quad y_1'(1) = 8, \quad y_2(1) = 1, \quad y_2'(1) = 3.$$

Compute their Wronskian $W(y_1, y_2)(t)$ as a function of t .