MATH 251

Examination

July 16, 2009

Name:____________________

Student Number:____________________

Section:____________________

This exam has 9 questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown. Credits will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM. PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE.

The last sheet of the booklet can be removed. **Be careful to remove only the last page of the examination**.

Do not write in this box.

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Total:________

1. (10 points) For parts (a) through (e) below, a list of differential equations is given. For each part, write down the letter corresponding to the equation on the list with the specified properties. **There is only one correct answer to each part.**

A. $y' = 2y + t$

B. $y' = e^{2y-t}$

C. $y' = e^y - 1$

D. $y'' + 4y' - 5y = 2$

E. $y'' + e^t y' + t^2 y = 0$

F. $y'' - 4y = \frac{t}{y}$

G. $y''' + 3y'' + 3y' + y = t^5 + \ln t$

H. $y''' + y'y = e^{-2t} \sin 5t$

(a) Second order homogeneous linear equation.

(b) Third order nonlinear equation.

(c) Second order nonhomogeneous linear equation.

(d) First order linear equation.

2. (10 points) Solve explicitly for $y(t)$ in the following initial value problem

$$e^t - y\,y' = 0; \qquad y(0) = 1.$$

3. (10 points) A tank is filled with 200 liters of a solution containing 100 grams of salt. A solution containing a concentration of 2 g/liter salt enters the tank at the rate 4 liters/minute and the well-stirred mixture leaves the tank at the same rate. Set up the initial value problem for the amount of salt in the tank at time t, find the particular solution and find the limiting amount of salt in the tank as $t \to \infty$.

4. (15 points) For the following initial value problem $ty' = 3y + t; y(4) = -1$

   (a) Without solving it, find the maximum interal on which we are guaranteed that the problem has a uniqe solution.

   (b) Solve the initial value problem.

5. (15 points) Find the particular solution to $y'' - 5y' + 4y = 0$, $y(0) = 2$, $y'(0) = -1$. What is the behavior of the solutions when $t \to +\infty$?

6. (5 points) Show $y_1(t) = t^2$ and $y_2(t) = t^3, t > 0$ are linearly independent by calculating the Wronskian.

7. (10 points) Provided $y_1(t) = t$ solves the equation $t^2y'' + 2ty' - 2y = 0$, $t > 0$, write down the general solution of the above equation.

8. (10 points) What is the form of the general solution to the following equation?

   Do <u>not</u> solve for the constants!

$$y'' - y = \cos(2t) + 3te^t - 4\sin(t)$$

9. (15 points) (a) Circle the correct answer. By definition $\{f(t)\} =$

(i) $\int_0^\infty e^{st} f(t)dt$

(ii) $\int_0^\infty e^{-st} f(t-c)dt$

(iii) $\int_0^\infty e^{-st} f(t)dt$

(b) Solve the following equation using Laplace's transform.

$$y' - y = e^t, \qquad y(0) = 1.$$

No credits will be given for other methods.

Table of Laplace Transform

$f(t)$	$F(s)$	comments
c	$\frac{1}{s}$	$s > 0$
t^n	$\frac{n!}{s^{n+1}}$	
e^{at}	$\frac{1}{s-a}$	$s > a$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$s > 0$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$s > 0$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	$n =$ positive integer, $s > a$