

MATH 251
Examination I
July 9, 2007

Name: _____
Student Number: _____
Section: _____

This exam has 11 questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM.

Do not write in this box.

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Total: _____

1. (10 points) For parts (a) through (e) below, a list of differential equations is given. For each part, write down the letter corresponding to the equation on the list with the specified properties. There is only one correct answer to each part.

A. $y' = 2y + t$

B. $y' = e^{2y-t}$

C. $y' = e^y - 1$

D. $y'' + 4y' - 5y = 2$

E. $y'' + e^t y' + t^2 y = 0$

F. $y'' - 4y = \frac{t}{y}$

G. $y''' + 3y'' + 3y' + y = t^5 + \ln t$

H. $y''' + y'y = e^{-2t} \sin 5t$

(a) First order autonomous equation.

(b) Second order homogeneous linear equation.

(c) Third order nonlinear equation.

(d) Second order nonhomogeneous linear equation.

(e) First order linear equation.

2. (5 points) Consider the initial value problem

$$(t - 4)y' + \frac{1}{t^2}y = \frac{e^t}{t + 2}, \quad y(2) = -1.$$

Without solving the equation, what is the largest interval in which a unique solution is guaranteed to exist?

- (a) $(-2, 0)$
 - (b) $(0, 4)$
 - (c) $(0, \infty)$
 - (d) $(-\infty, 0)$
3. (5 points) All of the equations below have $y(t) = 5e^{6t}$ as a particular solution, EXCEPT

- (a) $y'' - 12y' + 36y = 0$
- (b) $y'' - 5y' - 6y = 0$
- (c) $y'' + 4y' - 12y = 0$
- (d) $y'' - 36y = 0$

4. (5 points) Consider all the nonzero solutions of

$$y'' + 4y' + 3y = 0.$$

How will the solutions behave as $t \rightarrow \infty$?

- (a) They all go to 0.
 - (b) They all go to $-\infty$.
 - (c) They all go to ∞ .
 - (d) Some go to ∞ , the others go to $-\infty$.
5. (5 points) Let $y_1(t)$ and $y_2(t)$ be any two solutions of the second order linear equation

$$ty'' + 4y' + \sin(2t)y = 0.$$

In what form must their Wronskian, $W(y_1, y_2)(t)$, appear?

- (a) $\frac{C}{t^4}$
- (b) Ce^{4t}
- (c) Ct^4
- (d) Ce^{-4t}

6. (5 points) Consider the initial value problem

$$y' = \frac{4x^3 + 1}{2y - 4}, \quad y(0) = 0.$$

Which of the following statements is false?

- (a) The equation is separable.
- (b) The equation is exact.
- (c) The implicit form of its solution is $y^2 - 4y = x^4 + x$.
- (d) The explicit form of its solution is $y = 2 + \sqrt{x^4 + x + 4}$.

7. (14 points) Consider the autonomous equation

$$y' = -y^3 + y^2 + 2y = -y(y - 2)(y + 1)$$

(a) Find all equilibrium solutions.

(b) Classify the stability of each equilibrium solution. Justify your answer.

(c) If $y(\pi) = 0$, find $y(t)$. (You do **not** need to solve the equation.)

(d) If $y(5) = -2$, what is $\lim_{t \rightarrow \infty} y(t)$?

(e) If $y(-1) = \alpha$. For what value (or range of values) of α would $\lim_{t \rightarrow \infty} y(t) = 2$?

8. (12 points) Given the initial value problem

$$(6xy^2 + \cos y - 3x^2) dx + (6x^2y - x \sin y) dy = 0, \quad y(2) = 0.$$

(a) Verify that the equation is an exact equation.

(b) Solve the initial value problem.

9. (12 points) Solve the following initial value problem

$$ty'' + y' = 0, \quad t > 0, \quad y(1) = 2, \quad y'(1) = 1.$$

Hint: Use the substitutions $u = y'$ and $u' = y''$ to convert the equation into a first order linear equation in terms of u . Then integrate your answer to find y and lastly apply the initial conditions.

10. (12 points) Find the general solution of the nonhomogeneous linear equation

$$y'' - 2y' + 5y = 5t^2 + 6t - 12.$$

11. (15 points) A culinary experiment that went horribly awry has filled a $60 m^3$ kitchen with air that contains $2 g/m^3$ of smoke and soot. At $t = 0$, the ventilation system is switched on so that $3 m^3/min$ of fresh air is pumped in. The well-mixed smokey air is drawn off at the same rate.
- (a) Let $Q(t)$ denote the amount of smoke and soot in the air at any time $t > 0$. Write down an initial value problem (be sure to give both an equation and an initial condition) that $Q(t)$ must satisfy.
- (b) Solve the initial value problem to find $Q(t)$.
- (c) How much time would it take for the concentration of smoke and soot in the air to go down to $1/10$ of its original level?