

MATH 251  
Summer 2003  
Exam 1  
July 2, 2003

NAME : \_\_\_\_\_

ID : \_\_\_\_\_

INSTRUCTOR : \_\_\_\_\_

There are 9 questions on 8 pages.

Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice.

For partial credit questions, all work must be shown - **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.  
PLEASE DO NOT WRITE IN THE BOX BELOW.

1: _____
2: _____
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7: _____
8: _____
9: _____
Total: _____

1. (5 points) Which of the following is a second order, linear, homogeneous differential equation?

(a)  $y'' + y^2 = 0$

(b)  $y'' + y' + 2y = t \ln y$

(c)  $ty'' + y = 0$

(d)  $(y')^2 - t^2y = 1$

2. (5 points) Which pair of functions below is linearly independent?

(a)  $e^t, -e^t$

(b)  $\sin 3t, 0$

(c)  $e^{2t}, e^{2t+1}$

(d)  $e^{-t}, 2 + e^{-t}$

3. (5 points) Which equation has  $y_1 = e^{-2t}$  and  $y_2 = e^{3t}$  as two solutions?

(a)  $-2y' + 3y = 0$

(b)  $y'' + y' - 6y = 0$

(c)  $-y'' + y' + 6y = 0$

(d)  $2y'' + 10y' - 12y = 0$

4. (15 points)

(a) Solve the initial value problem:  $ty' + 2y = 3t$ ,  $y(2) = 5$ .

(b) What is the largest interval on which the solution is guaranteed to exist?

5. (12 points) For the initial value problem:

$$4x^3 + 2y + y \cos x + (2x + \sin x + 2y) \frac{dy}{dx} = 0, \quad y(0) = 2,$$

- (a) Verify that the equation is exact.
- (b) Solve the initial value problem.

6. (16 points) Consider the autonomous equation:

$$y' = y^2(y^2 - 100).$$

- (a) Find the equilibrium solutions.
- (b) For each equilibrium solution, classify its stability. Justify your answer.
- (c) If  $y(0) = 1.75$ , what is  $\lim_{t \rightarrow \infty} y(t)$ ?
- (d) If  $y(10) = 0$ , what is  $y(100)$ ?

7. (18 points) A  $200 \text{ m}^3$  room initially contains fresh air. At  $t = 0$ , a faulty heating system causes gas containing 20% carbon monoxide to be pumped into the room at a rate of  $3 \text{ m}^3$  per minute. The well-mixed air is vented out at the same rate.
- (a) Write a differential equation, and give the initial condition, that describe this event.
  - (b) Solve the initial value problem.
  - (c) A carbon monoxide detector in the room is triggered when the carbon monoxide reaches 1%. Find the time when the detector will sound the alarm.

8. (12 points) Solve the initial value problem:

$$y'' - 2y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = -4.$$

9. (12 points) Find the general solution of  $2t^2y'' - ty' + y = 0, t > 0$ , given that  $y_1(t) = t$  is a known solution.