

MATH 251  
Summer 2002  
Exam 1  
June 27, 2002

NAME : \_\_\_\_\_

ID : \_\_\_\_\_

INSTRUCTOR : \_\_\_\_\_

There are **10** questions on **9** pages. Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice. For partial credit questions, all work must be shown - **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.  
PLEASE DO NOT WRITE IN THE BOX BELOW.

1: _____
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9: _____
10: _____
Total: _____

1. (6 points) Which of the following is a linear second order differential equation?

(a)  $y' + ty = 1$

(b)  $y'' = t^2y + e^t$

(c)  $(y')^2 = (y + 2)(y - 3)$

(d)  $y'' + 3y' + 2y = \sin y$

2. (6 points) The Existence and Uniqueness Theorem guarantees that the solution to

$$(t^2 - 4)y' + 2ty = \frac{1}{t^2} \quad y(1) = 4$$

is valid on;

(a)  $(-2, 2)$

(b)  $(0, 2)$

(c)  $(2, \infty)$

(d)  $(-\infty, \infty)$

3. (6 points) Which of the following pairs of functions is linearly dependent?

- (a)  $\sin 5t, \cos 5t$ .
- (b)  $e^{2t} \sin t, e^{2t} \cos t$ .
- (c)  $e^{2t}, e^{4t}$ .
- (d)  $e^t, e^{t+4}$ .

4. (6 points) Which of the following is the general solution of

$$y'' + 2y' + 5y = 0$$

- (a)  $c_1 e^t + c_2 e^{5t}$ .
- (b)  $c_1 e^{2t} \cos t + c_2 e^{2t} \sin t$ .
- (c)  $c_1 e^{-t} \cos 4t + c_2 e^{-t} \sin 4t$ .
- (d)  $c_1 e^{-t} \cos 2t + c_2 e^{-t} \sin 2t$ .

5. (12 points) Solve **explicitly** the initial value problem;

$$y' = \frac{3x^2 + 4x + 2}{2y - 6} \quad y(0) = 4$$

6. (14 points) A 400-liter tank is initially filled with 100 liters of dye solution with a dye concentration of 5 g/l. Pure water flows into the tank at a rate of 3 liters per minute. The well-stirred solution is drained at a rate of 2 liters per minute. Find the concentration of dye in the tank at the time that the tank is completely filled.

7. (12 points) Consider the autonomous equation

$$y' = y^3 - 16y$$

- (a) Find the equilibrium solutions.
- (b) Classify the stability at each equilibrium solution. You must justify your answer.
- (c) If  $y(1) = 4$ , what is  $\lim_{t \rightarrow \infty} y(t)$ ?

8. (12 points) Solve the initial value problem;

$$\left(2x + 4xy + \frac{y}{x}\right)dx + (2x^2 + \ln x + 3y^2)dy = 0 \quad y(1) = -1$$

9. (14 points)

(a) Solve the initial value problem:  $y'' - y' - 6y = 0$ ,  $y(0) = \alpha$ ,  $y'(0) = 4$ .

(b) If  $\lim_{t \rightarrow \infty} y(t) = 0$ , what is the value of  $\alpha$ ?

10. (12 points) Given that  $y_1(t) = t^3$  is a solution, find the general solution of

$$t^2 y'' + 2ty' - 12y = 0 \quad t > 0$$