Name_____    ID #_____    Section #_____

Instructor_____

Show all your work for each of the partial-credit problems.

**Points awarded**

1. (16 pts)                                _____

2. (17 pts)                                _____

3. (17 pts)                                _____

4. (17 pts)                                _____

5. (16 pts)                                _____

6. (17 pts)                                _____

_____

**Total Points:** _____

1. (16 points)

a) Find the general solution of the equation

$$y' = 4y^2(x - x^3).$$

b) Find the general solution of the differential equation

$$xy' + y = xe^x$$

2. (17 points) There are 200 liters of pure water in a tank. Water containing 50 grams/liter of dye is entering the tank at a rate of 2 liters/min and the mixture is allowed to flow out of the tank at the same rate.

    a) Find the quantity of dye at any time t. (Write down the equation that describes the process and solve it. Show all your work.)

    b) Find the limit value of the quantity when $t \to \infty$.

    c) Find the time when the quantity of dye in the solution reaches 50% of its limit value.

3. (17 points) A ten kilogram ball is thrown down from a cliff with an initial velocity of 1 meter per second. Air resistance acts on the ball with a force equal to 7 times the speed. If the acceleration due to gravity is 9.8 meters/second$^2$, find:

   a) The velocity at time t.

   b) The distance fallen by time t.

   c) The limiting velocity as $t \to \infty$.

4. (17 points) Solve the following initial value problem:

$$y'' - 4y' + 20y = 0; \quad y(2) = 0, \quad y'(2) = 8$$

5. (16 points) Solve the following initial value problem:

$$y'' + y' - 12y = 0; \quad y(0) = 3, \quad y'(0) = -5$$

6. (17 points) Find a second, linearly independent, solution of the differential equation
$x^2 y'' - (x^2 + 2x)y' + (x+2)y = 0$ given that $y_1 = xe^x$ is a solution.