MATH 251

Examination I

February 17, 2011

FORM A

Name:_____

Student Number:_____

Section:_____

This exam has 14 questions for a total of 100 points. Show all you your work! **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work. For other problems, points might be deducted, at the sole discretion of the instructor, for an answer not supported by a reasonable amount of work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM. PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE.

Do not write in this box.

1:_____
2:_____
3 thru 10:_____
11:_____
12:_____
13:_____
14:_____
Total:_____

1. (6 points) In each part below, determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

   (a) (2 points) $y''' + ty' + (\cos^2 y)t = t^3$

   (b) (2 points) $y' + t^2 = y$

   (c) (2 points) $(1 + y^3)y'' + ty' + y = e^t$

2. (6 points) Find a suitable integrating factor that could be used to solve the equation below.

$$\frac{1}{t^3}y' + \frac{e^t}{t^2}y = \sin(t), \qquad t > 0.$$

3. (5 points) Let $y(x)$ be the solution to the initial value problem

$$y' - 2y^2 = xy^2, \qquad y(0) = 1.$$

Which of the following values is $y(1)$?

(a) 0

(b) $\dfrac{4}{3}$

(c) -1

(d) $\dfrac{-2}{3}$

4. (5 points) A tank initially contains 200 gallons of water with a salt concentration of 0.5 oz/gal. Water containing a salt concentration of $2 + \sin(t)$ oz/gal flows into the tank at a rate of 5 gal/min, and the mixture in the tank flows out at the same rate. Let $Q(t)$ denote the amount of salt in the tank at time t. Which of the following initial value problems accurately describes the situation?

(a) $Q' = 10 + 5\sin(t) - \frac{Q}{40}, \qquad Q(0) = 100.$

(b) $Q' = \frac{2}{5} + \frac{1}{5}\sin(t) - Q, \qquad Q(0) = 100.$

(c) $Q' = 2 + \sin(t) - 5Q, \qquad Q(0) = 200.$

(d) $Q' = 10 + 5\sin(t) + \frac{Q}{40}, \qquad Q(0) = 200.$

5. (5 points) Consider the solution $y(t)$ of the initial value problem

$$y' - 2y = 2, \qquad y(0) = 2.$$

What is $\lim_{t\to\infty} y(t)$?

(a) $-\infty$

(b) -1

(c) 2

(d) ∞

6. (5 points) Consider the following autonomous differential equation

$$y' = \cos(y).$$

The function $y(t) = \dfrac{5\pi}{2}$ is

(a) a semi-stable equilibrium solution.

(b) an unstable equilibrium solution.

(c) an asymptotically stable equilibrium solution.

(d) not an equilibrium solution.

7. (5 points) Consider the initial value problem

$$(t+5)y'' + \frac{t+2}{t-\pi}\,y' + \ln(t)\,y = 0, \qquad y(1) = -10, \qquad y'(1) = 3.$$

Without solving the equation, what is the largest interval in which a unique solution is guaranteed to exist?

(a) $(-\infty, \infty)$

(b) $(-5, \pi)$

(c) $(-2, \infty)$

(d) $(0, \pi)$

8. (5 points) Find the general solution of

$$16y'' + 8y' + y = 0.$$

(a) $y = C_1 e^{t/4} + C_2 e^{t/4}$

(b) $y = C_1 e^{-t/4} + C_2 t e^{-t/4}$

(c) $y = C_1 e^{-4t} + C_2 e^{4t}$

(d) $y = C_1 e^{-t/4} + C_2 e^{-4t}$

9. (5 points) Which pair of functions below can be a fundamental set of solutions?

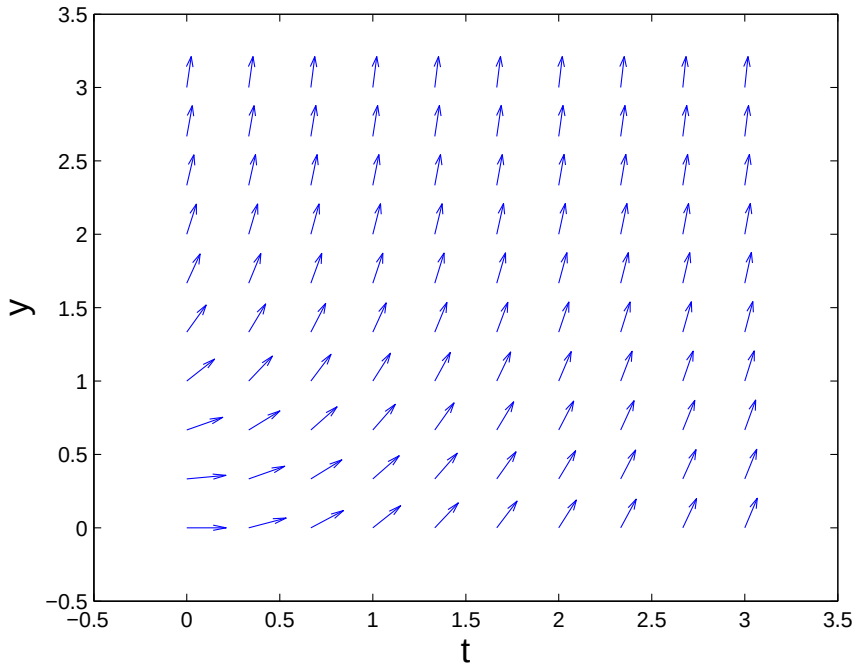
(a) $1-2t, \qquad 4t-2$

(b) $3\cos(t), \qquad -2\sin(t)$

(c) $-e^{2t}, \qquad e^{2t+5}$

(d) $0, \qquad 3\sin(2t)$

10. (5 points) Determine the differential equation whose direction field is given below.



(a) $y' = y^2$

(b) $y' = t$

(c) $y' = y^2 + t$

(d) $y' = y^2 - t$

11. (12 points)

(a) (4 points) Consider the differential equation

$$(2\alpha y^2 - 2ye^{2x} + 4) + (2xy - e^{2x} - 3\alpha y^2)\, y' = 0.$$

Find the value α that would make this equation an exact equation.

(b) (8 points) Given that the differential equation

$$(6x^2y^4 + y\sec^2(xy) + 2x) + (8x^3y^3 + x\sec^2(xy) - 1)y' = 0$$

is an exact equation, find the solution of the equation that also satisfies the initial value $y(0) = -6$. You may leave your answer in an implicit form.

12. (12 points) Consider the initial value problem

$$y'' - 7y' + 12y = 0, \qquad y(0) = 4, \qquad y'(0) = 11.$$

(a) (9 points) Find the solution $y(t)$ of this initial value problem.

(b) (3 points) Determine $\lim_{t\to\infty} y(t)$.

13. (12 points) Given that $y_1(t) = t$ is a known solution of the second order linear differential equation

$$t^2y'' - 4ty' + 4y = 0, \qquad t > 0.$$

Find the general solution of the equation.

14. (12 points) Find the general solution of the nonhomogeneous second order linear equation

$$y'' - 4y' + 5y = e^{2t} - 10t.$$