

MATH 251
Midterm Exam I
Spring 2003

NAME _____
STUDENT NUMBER _____
INSTRUCTOR _____

There are **11** questions on **9** pages. Please read each problem carefully before starting to solve it. Show all work – **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.

Point - grader use only

1. (8 pt) _____
2. (6 pt) _____
3. (6 pt) _____
4. (6 pt) _____
5. (6 pt) _____
6. (10 pt) _____
7. (14 pt) _____
8. (10 pt) _____
9. (12 pt) _____
- 10.(12 pt) _____
- 11.(10 pt) _____
- Total _____

1. (8 points) Classify the following differential equations as linear or non-linear and state their order.

	linear/ non-linear	order
$\ln(t) \frac{d^2y}{dt^2} + 3e^t \frac{dy}{dt} - y \sin t = 0$		
$2y' - y^2 = e^t$		
$y''' + (t^2 - 1)y + \cos t = 0$		
$y'' - \sin(t + y)y' + (t^2 + 1)y = 0$		

2. (6 points) What is the integrating factor of the differential equation

$$t^2 y'' - 4ty = e^t - \cos 2t$$

- (a) $\mu(t) = 1/t^4$
(b) $\mu(t) = e^{4 \ln t}$
(c) $\mu(t) = -t^4$
(d) $\mu(t) = -2t^2$

3. (6 points) The Existence and Uniqueness Theorem guarantees that the solution to

$$(t + 2)y'' - \sin ty' + \frac{ty}{t - 4} = \frac{e^{2t}}{t}, \quad y(-1) = 0$$

- (a) is valid on $(-\infty, \infty)$
(b) is valid on $(-\pi, 0)$
(c) is valid on $(-2, 0)$
(d) does not exist
4. (6 points) Which of the equations below has $y(t) = c_1e^t + c_2e^{-2t}$ as its general solution?

- (a) $y' - 2y = 0$
(b) $y'' - y' - 2y = 0$
(c) $2y'' + 2y' - 4y = 0$
(d) $y'' - 3y' - 2y = 0$

5. (6 points) What is the form of the particular solution of

$$y'' + 4y' + 4y = te^{-2t} + 2t^2 \cos 2t - 3$$

- (a) $y(x) = (At^3 + Bt^2)e^{-2t} + (Ct^2 + Dt + E) \cos 2t$
(b) $y(x) = (At^2 + Bt)e^{-2t} + (Ct^2 + Dt + E) \cos 2t + F$
(c) $y(x) = (At + B)e^{-2t} + (Ct^2 + Dt + E) \cos 2t + (Ft^2 + Gt + H) \sin 2t + I$
(d) $y(x) = (At^3 + Bt^2)e^{-2t} + (Ct^2 + Dt + E) \cos 2t + (Ft^2 + Gt + H) \sin 2t + I$

6. (10 points) Let $y' = -y(y^2 - 4)$.

(a) Find all equilibrium solutions.

(b) Determine the stability of each equilibrium solution. Justify your answer.

(c) If $y(5) = 1$, what is $\lim_{t \rightarrow \infty} y(t)$?

7. (14 points) Certain bacteria has a volume of $2nl$ which we assume to be mostly cytoplasm. This bacteria is placed in a substance polluted with $3ng/nl$ of certain harmful chemical. The bacteria exchanges fluids with its media at a rate of $1/100nl/s$, that is there is a flow of $1/100nl/s$ of the polluted substance into the bacteria, and the (well mixed) cytoplasm flows out of the bacteria at the same rate. Assume that the bacteria is initially clean from the chemical.

(a) Find a formula for the amount of the chemical in the cell at any given time.

(b) The bacteria is going to die when the concentration of the chemical reaches $3ng/nl$. When is this going to happen?

8. (10 points) Consider the differential equation:

$$(\ln(y) + 2xy + 2 \cos 2x) + \left(\frac{x}{y} + x^2\right)y' = 0$$

(a) Show that the above equation is exact.

(b) Give the general solution to the equation.

9. (12 points) (a) Are the functions $y_1(t) = t^3$ and $y_2(t) = 1/t$ solutions to the differential equation

$$t^2 y'' - t y' - 3y = 0$$

- (b) Are the functions y_1 and y_2 linear independent? Justify your answer.

- (c) What is the general solution of the differential equation of part (a). Justify.

10. (12 points) (a) Find the solution to the homogeneous differential equation

$$y'' + 3y' - 4y = 0$$

(b) Find the general solution to

$$y'' + 3y' - 4y = 50 \sin 2t$$

11. (10 points) Solve the initial value problem

$$y'' + 6y' + 9y = 0, \quad y(0) = 3, \quad y'(0) = -1$$