MATH 251
1st exam
Feb 21, 2000

Name:_____
Student Number:_____
Instructor:_____
Section:_____

There are **8** partial credit questions. **In order to obtain full credit for these problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.
THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION. At the end of the examination, the booklet will be collected.

1:_____	
2:_____	
3:_____	
4:_____	
5:_____	
6:_____	
7:_____	
8:_____	**Do not write in the box to the left**
Total:_____	

1. (10 points) Classify the following equations as linear or non-linear, homogeneous or non-homogeneous, ordinary or partial, and by their order.
   (a) $\frac{dy}{dt} = ty^2$
   (b) $y'' + y = t - 2$
   (c) $(1 - y)\frac{d^2y}{dt^2} - 2\frac{dy}{dt} = \cos 2t$
   (d) $3x'' + 2x' - 4x = 0$

2. (15 points) A tank originally contains 100 litres of pure water. Salt water with a concentration of 2 kg/L is pumped into the tank at 3 L/min, and the well-mixed solution is drained at the same rate.
   (a) Set up an initial value problem describing the situation. Be sure to explain all of your variables.
   (b) Solve the intial value problem to find the amount of salt in the tank at any time t
   (c) What is the limiting concentration of salt in the tank?

3. (10 points) Find the solution of the given initial value problem and state the largest interval in which the solution is guaranteed to be valid.

   $ty' + 2y = \sin t \qquad y(\frac{\pi}{2}) = 1.$

4. (a) (10 points) Solve the initial value problem

$$y' - \frac{1}{2}y = e^{-t} \qquad y(0) = y_0.$$

(b) Find the value of y_0 which separates solutions that grow positively from those that grow negatively as $t \to \infty$. What is the asymptotic behavior of the solution for this critical value of y_0?

5. (15 points)

   Find the general solution for each of the following equations.

   (a) $y'' - y' - 2y = 0$

   (b) $y'' + 4y' + 4y = 0$

   (c) $y'' + 4y' + 5y = 0$

6. (10 points) A 2 kg mass is placed on a spring with $k = 8$. At $t = 0$, the system is suddenly set in motion from its equilibrium position by an external force given by $2\cos(\omega t)$, where ω is a positive constant. For which value of ω will the system have resonance?

7. (15 points) Suppose a population obeys the equation

$$\frac{dy}{dt} = -r(1 - \frac{y}{T})(1 - \frac{y}{K})y \quad r > 0, \quad 0 < T < K, \quad y(0) = y_0$$

(a) Find the equilibrium solutions and determine their stability.

(b) For what values of $y_0 > 0$ will the population die out? Become stable? Increase without bound?

8. (15 points) What is the *form* of a particular solution of the equation:

$$y'' - 6y' + 9y = t^2 + e^{3t} - 4\cos t$$

Do not solve for the constants.