MATH 251

Examination I

October 8, 2015

FORM A

Name:________________

Student Number:________________

Section:________________

This exam has 14 questions for a total of 100 points. Show all you your work! **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work. For other problems, points might be deducted, at the sole discretion of the instructor, for an answer not supported by a reasonable amount of work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM. PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE.

Do not write in this box.

1 through 9:		(45)
10:		(8)
11:		(12)
12:		(9)
13:		(14)
14:		(12)
Total:		

1. (5 points) What is a suitable integrating factor that can be used to solve the equation

$$ty' - (t+2)y = 2\tan(3t)?$$

(a) $\mu(t) = \dfrac{1}{t^2 e^t}$

(b) $\mu(t) = e^{-\frac{t+2}{t}}$

(c) $\mu(t) = e^{\frac{t^2}{2} - 2t}$

(d) $\mu(t) = t^2 e^t$

2. (5 points) Consider the initial value problem

$$(t^2 - 4)y' + t^2 y = e^t, \qquad y(-3) = 4.$$

According to the Existence and Uniqueness Theorem, what is the largest interval in which a unique solution is guaranteed to exist?

(a) $(2, \infty)$

(b) $(-2, 2)$

(c) $(-\infty, -2)$

(d) $(-\infty, 0)$

3. (5 points) Which of the following functions is a solution of the differential equation

$$e^y t^{-2} y' - 3 = 0?$$

(a) $y(t) = e^{3t}$

(b) $y(t) = 3\ln t$

(c) $y(t) = t^3$

(d) $y(t) = \dfrac{2}{t}$

4. (5 points) A mixing tank initially contains 50 lb of salt dissolved in 900 gal of water. Water containing $2 + \cos(\pi t)$ lb/gal of salt enters the tank at a rate of 3 gal/min. The well-stirred mixture flows out of the tank at the same rate. Let $Q(t)$ be the amount of salt in the tank at time t. Which of the following initial value problems accurately describes $Q(t)$, for $t \geq 0$?

(a) $\dfrac{dQ}{dt} = -3(2 + \cos(\pi t)) + \dfrac{Q}{300}, \qquad Q(0) = 900.$

(b) $\dfrac{dQ}{dt} = 3(2 + \cos(\pi t)) - \dfrac{Q}{300}, \qquad Q(0) = 900.$

(c) $\dfrac{dQ}{dt} = -3(2 + \cos(\pi t)) + \dfrac{Q}{300}, \qquad Q(0) = 50.$

(d) $\dfrac{dQ}{dt} = 3(2 + \cos(\pi t)) - \dfrac{Q}{300}, \qquad Q(0) = 50.$

5. (5 points) The velocity, given in meters per second, of a certain particle is given by the initial value problem

$$\frac{dv}{dt} = 1000 - \frac{1}{90}v^2, \qquad v \geq 0, \qquad v(0) = 5.$$

Approximately how fast will the particle be moving after *a very long time*?

(a) $\infty\ m/s$

(b) $300\ m/s$

(c) $90\ m/s$

(d) $0\ m/s$

6. (5 points) Suppose $y_1(t) = e^{-\pi t}$ and $y_2(t) = e^{\sqrt{2}t}$ are two solutions of a certain second order differential equation

$$y'' + p(t)y' + q(t)y = 0.$$

Which of the following statements is **false**?

(a) $y(t) = 0$ is another solution.

(b) $y(t) = \sqrt{2}e^{-\pi t} - \pi e^{\sqrt{2}t}$ is another solution.

(c) $y(t) = e^{(\sqrt{2}-\pi)t}$ is another solution.

(d) $y_1(t)$ and $y_2(t)$ form a pair of fundamental solutions.

7. (5 points) Let $y(t)$ be the solution of the initial value problem

$$y'' - y' - 20y = 0, \qquad y(0) = 2, \qquad y'(0) = \beta.$$

Find the value of β for which $\lim_{t\to\infty} y(t) = 0$.

(a) 0.

(b) -8

(c) 10

(d) -2

8. (5 points) Suppose $y_1(t)$ and $y_2(t)$ are two solutions of a certain second order linear differential equation

$$4t^2y'' + 2ty' - 5t^3y = 0.$$

Which function below could possibly be their Wronskian, $W(y_1, y_2)(t)$?

(a) $W(y_1, y_2)(t) = \sqrt{t}$

(b) $W(y_1, y_2)(t) = 2e^{-t^2}$

(c) $W(y_1, y_2)(t) = \dfrac{3}{\sqrt{t}}$

(d) $W(y_1, y_2)(t) = 4e^{t^2}$

9. (5 points) Find the general solution of the fourth order linear equation

$$y^{(4)} + 6y''' + 10y'' = 0.$$

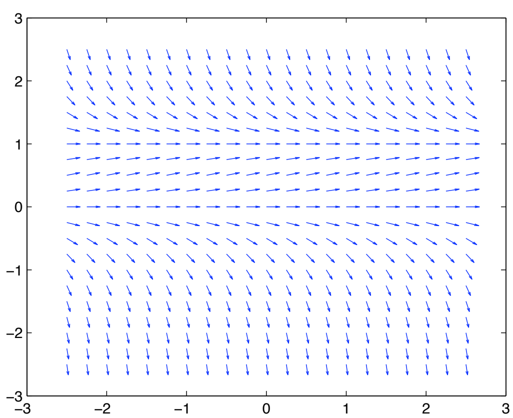
(a) $y(t) = C_1 + C_2 t + C_3 e^{-3t}\cos t + C_4 e^{-3t}\sin t$

(b) $y(t) = C_1 + C_2 t + C_3 e^{3t}\cos t + C_4 e^{3t}\sin t$

(c) $y(t) = C_1 + C_2 t + C_3 e^{t}\cos 3t + C_4 e^{t}\sin 3t$

(d) $y(t) = C_1 t + C_2 t^2 + C_3 e^{-3t}\cos t + C_4 e^{-3t}\sin t$

10. (8 points) Consider the following direction field of a certain first order equation:



(a) (4 points) Which of the equations below does it represent?

(A) $y' = y - x$

(B) $y' = y + x$

(C) $y' = y(1 - y)$

(D) $y' = x(1 - x)$

(b) (4 points) There are two equilibrium solutions shown in the direction field. Identify them and classify the stability of each.

11. (12 points) Consider the differential equation

$$(y\cos(x) + ye^{xy}) + (\sin(x) + xe^{xy})y' = 0.$$

(a) (3 points) Verify that it is an exact equation.

(b) (2 points) Is this equation also separable?

(c) (7 points) Find the solution of this equation satisfying $y(0) = 2$. You may leave your answer in an implicit form.

12. (9 points) Consider the second order linear equation

$$y'' - 10y' + 25y = 0.$$

(a) (3 points) Find the general solution of the equation.

(b) (4 points) Find the solution satisfying the initial conditions $y(0) = 1,\ y'(0) = 2$.

(c) (2 points) For your answer from (b), determine $\lim_{t\to\infty} y(t)$.

13. (14 points) Consider the second order nonhomogeneous linear equation

$$y'' - 4y' = 5 - e^{2t}.$$

(a) (3 points) Find $y_c(t)$, the solution of its corresponding homogeneous equation.

(b) (8 points) Find its general solution by using the Method of Undetermined Coefficients.

(c) (3 points) What is the **form** of particular solution Y that you would use to solve the following equation using the Method of Undetermined Coefficients? **DO NOT ATTEMPT TO SOLVE THE COEFFICIENTS.**

$$y'' - 4y' = t^2 + 2te^{4t} \sin t.$$

14. (12 points) Consider a mass-spring system described by the equation

$$3u'' + \gamma u' + ku = 0, \qquad \gamma \geq 0, \qquad k > 0.$$

Answer the following questions. Be sure to justify your answer. Full credit will not be given without supporting work.

(a) (2 points) Suppose the spring was stretched 6 meters by the mass to its equilibrium position. Find the value of k. You may use $g = 10$ as the gravitational constant.

(b) (2 points) Suppose $k = 10$. For what value(s) of γ would the system be *critically damped*?

(c) (2 points) Suppose $\gamma = 15$ and $k = 12$. Will any nonzero solution of the equation cross the equilibrium position exactly once?

(d) (2 points) Suppose $\gamma = 6$ and $k = 15$. Find the *quasi frequency* of the system.

(e) (4 points) Suppose a force of $F(t) = 49\sin(\omega t)$ is applied to the system and given $k = 192$. What are the values of γ and ω if the system exhibits resonance?