MATH 251 Name:________________

Examination I Student Number:________________

October 2, 2008 Section:________________

This exam has 11 questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM. PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE.

Do not write in this box.

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1. (12 points) For parts (a) through (d) below, a list of differential equations is given. For each part, write down the letter corresponding to the equation on the list with the specified properties. There is only one correct answer to each part. Afterward also answer part (e).

A. $y' = 2y^3 - 16$

B. $y'' - 2y' + 10y = 0$

C. $y' + ty = e^{-t}\sin t$

D. $y'' + 2t^2y' + 9y = te^{-4t}$

E. $y' = \dfrac{6y}{t}$

F. $y''' - y^2 = 1$

G. $y'' + 5y' + y = \tan(2y)$

(a) (2 points) First order linear equation that is also separabe.

(b) (2 points) First order nonlinear, autonomous, equation.

(c) (2 points) Second order nonlinear equation.

(d) (2 points) Second order nonhomogeneous linear equation.

(e) (4 points) What is a suitable integrating factor that could be used to solve the linear equation you found in part (a)?

2. (5 points) Consider the initial value problem

$$(t^2 - 4)y' + \frac{t+2}{t}\,y = \frac{t^3}{t-5}, \qquad y(4) = \frac{1}{2}.$$

Without solving the equation, what is the largest interval in which a unique solution is guaranteed to exist?

(a) $(0, 2)$

(b) $(0, 5)$

(c) $(2, 5)$

(d) $(2, \infty)$

3. (5 points) Which pair of functions below <u>cannot</u> be a fundamental set of solutions?

(a) $e^{2t} - 1, \qquad 2 - 2e^{2t}$

(b) $2\cos 4t, \qquad 3\sin 4t$

(c) $2e^{5t}, \qquad e^{5t} - 5$

(d) $1, \qquad e^{-2t}$

4. (5 points) Find the general solution of the exact equation

$$xy^2 + x^2 + (x^2y + y)\,y' = 0.$$

(a) $2xy = C$

(b) $3x^2y^2 + 2x^3 = C$

(c) $x^2y^2 + y^2 = C$

(d) $\dfrac{x^2y^2}{2} + \dfrac{x^3}{3} + \dfrac{y^2}{2} = C$

5. (5 points) A college student currently owes \$1000 on his credit card balance that carries an interest rate of 15% per year compounded continuously. Suppose the student's spending habit causes his credit card balance to increase continuously, net of his monthly payment, by \$100 every month (a total of \$1200 each year). Which initial value problem below describes his credit card balance, as a function of time t onward, up to the moment when his credit limit is reached?

(a) $y' = 0.15y + 100, \qquad y(0) = 1000.$

(b) $y' = 0.15y + 1200, \qquad y(0) = 1000.$

(c) $y' = 0.15y - 100, \qquad y(0) = 1000.$

(d) $y' = 0.15y - 1200, \qquad y(0) = 1000.$

6. (5 points) A tank whose capacity is 200 liters contains a mixture consisting of 300 grams of salt and 100 liters of water. At 8:00 am mixture with a concentration of 3 grams/liter begins to flow into the tank at the rate of 4 liters/min. Also, thoroughly mixed mixture is allowed to flow out of the bottom of the tank at the rate of 4 liters/min. Determine the quantity of salt that would be in the mixture in the tank after a very long time (that is, as $t \to \infty$).

   (a) 0 gram

   (b) 100 grams

   (c) 300 grams

   (d) 600 grams

7. (16 points) Consider the autonomous equation

$$y' = 9y^2 - y^4 = y^2(3 - y)(3 + y).$$

Answer the following questions without solving the equation.

(a) (3 points) Find all equilibrium solutions.

(b) (6 points) Classify the stability of each equilibrium solution. Justify your answer.

(c) (2 points) If $y(22/7) = \pi$, what is $\lim_{t\to\infty} y(t)$?

(d) (2 points) If $y(2\pi) = -3$, find $y(t)$.

(e) (3 points) If $y(4) = \lambda$. For what value (or range of values) of λ would $\lim_{t\to\infty} y(t) = 0$?

8. (10 points) Solve, explicitly for y as a function of x, the initial value problem

$$y' = \frac{9x^2 + 2}{2y}, \qquad y(1) = -2.$$

9. (15 points) In each of the parts (a), (b), and (c), find the real-valued general solution of the second order linear equation given.

(a) (3 points) $y'' + 12y' + 36y = 0$

(b) (3 points) $y'' - 7y' - 8y = 0$

(c) (3 points) $y'' - 4y' + 13y = 0$

For each of the parts (d) and (e), consider the solutions of the three equations above and write down the correct equation (equation (a), (b), or (c)) whose solutions behave as stated.

(d) (3 points) Every solution of this equation approaches 0 as $t \to \infty$.

(e) (3 points) Every **nonzero** solution of this equation does not approach a finite limit, nor does it have a limit of ∞ or $-\infty$, as $t \to \infty$.

10. (10 points) Given that $y_1(t) = \dfrac{1}{t^2}$ is a known solution of the second order linear differential equation

$$t^2y'' + ty' - 4y = 0, \qquad t > 0.$$

Find the general solution of this equation.

11. (12 points) Consider the nonhomogeneous second order linear equation of the form

$$y'' + 5y' + 4y = g(t).$$

(a) (3 points) Find its complementary solution, $y_c(t)$.

For each of the parts (b) through (d), write down the correct choice of the **form** of particular solution that you would use to solve the given equation using the Method of Undetermined Coefficients. **DO NOT ATTEMPT TO SOLVE THE COEFFICIENTS.**

(b) (3 points) $y'' + 5y' + 4y = 6e^{-4t} + 2e^{t}$

(c) (3 points) $y'' + 5y' + 4y = 2t^3e^{-t}$

(d) (3 points) $y'' + 5y' + 4y = t^2e^{-4t}\sin t$