MATH 251 Name:_____

Examination I Student Number:_____

October 4, 2007 Section:_____

This exam has 12 questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.

YOU MAY NOT USE A CALCULATOR ON THIS EXAM. PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE.

Do not write in this box.

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1. (5 points) Which function below is the integrating factor $\mu(t)$ that could be used to solve the first order linear differential equation

$$t^2y' - 4ty = 0?$$

(a) t^4

(b) e^{2t^2}

(c) $\dfrac{1}{t^4}$

(d) e^{-4t}

2. (5 points) Consider the first order differential equation

$$y' = -t^3.$$

Which statement below is **false**?

(a) The equation is linear.

(b) The equation is separable.

(c) The equation is exact.

(d) The equation is autonomous.

3. (5 points) Consider the initial value problem

$$(t+5)y'' + \frac{t+2}{t-\pi}y' + \ln(t)\, y = 0, \qquad y(1) = -10.$$

Without solving the equation, what is the largest interval in which a unique solution is guaranteed to exist?

(a) $(-\infty, -5)$

(b) $(-5, \pi)$

(c) $(-2, \infty)$

(d) $(0, \pi)$

4. (5 points) What is the general solution of

$$9y'' + 6y' + y = 0?$$

(a) $C_1\, e^{\frac{1}{3}t} + C_2\, e^{\frac{1}{3}t}$

(b) $C_1\, e^{-3t} + C_2\, e^{3t}$

(c) $C_1\, e^{\frac{-1}{3}t} + C_2\, te^{\frac{-1}{3}t}$

(d) $C_1\, e^{\frac{1}{3}t} + C_2\, e^{3t}$

5. (5 points) Let $y_1(t)$ and $y_2(t)$ be any two solutions of the second order linear equation

$$(t^2+4)y'' + 2ty' - t^3 y = 0$$

In what general form must their Wronskian, $W(y_1, y_2)(t)$, appear?

(a) $C(t^2+4)$

(b) $C\sqrt{t^2+4}$

(c) $\dfrac{C}{(t^2+4)}$

(d) $C(t^2+4)^2$

6. (5 points) A fish tank is initially filled with 400 liters of water containing $1\,g/liter$ of dissolved oxygen. At $t = 0$, oxygenated water containing $10\,g/liter$ of oxygen flows in at a rate of $4\,liter/min$. The well-mixed water is pumped out at a rate of $3\,liter/min$ from the tank. Which of the initial value problems below describes, $Q(t)$, the amount of dissolved oxygen in the tank at any time $t > 0$ (until the time when the tank, eventually, overflows)?

(a) $Q' = 40 - \dfrac{3}{400+t}Q, \qquad Q(0) = 400.$

(b) $Q' = 40 - \dfrac{3}{400+t}Q, \qquad Q(0) = 1.$

(c) $Q' = 40 - \dfrac{3}{400}Q, \qquad Q(0) = 10.$

(d) $Q' = 40 - \dfrac{3}{400-t}Q, \qquad Q(0) = 4000.$

7. (12 points) Consider the autonomous differential equation

$$y' = y(y-5)(10-y).$$

(a) (3 points) Find all equilibrium solutions.

(b) (5 points) Classify the stability of each equilibrium solution. Justify your answer.

(c) (2 points) If $y(5000) = 6$, what is $\lim_{t\to\infty} y(t)$?

(d) (2 points) If $y(7) = 10$, find $y(21)$. (You do **not** need to solve the equation to find the answer.)

8. (12 points)

(a) (4 points) Consider the differential equation

$$(\sin y + \alpha y) + (x \cos y + 2x + 2\alpha y^3)\, y' = 0.$$

Find the value α that would make this equation an exact equation.

(b) (8 points) Given that the differential equation

$$(5x^4y^2 + ye^{xy}) + (2x^5y + xe^{xy} - 4)y' = 0$$

is an exact equation, find the solution of the equation that also satisfies the initial value $y(0) = 5$. You may leave your answer in implicit form.

9. (10 points) Consider the initial value problem

$$y'' - 7y' + 6y = 0, \qquad y(0) = 5, \qquad y'(0) = 0.$$

(a) (8 points) Find the solution, $y(t)$, of this initial value problem.

(b) (2 points) What is $\lim_{t\to\infty} y(t)$?

10. (12 points) Consider the nonhomogeneous second order linear equation of the form

$$y'' - 4y' + 8y = g(t).$$

(a) (3 points) Find its complementary solution, $y_c(t)$.

For each of parts (b) through (d), write down the correct choice of the **form** of particular solution that you would use to solve the given equation using the Method of Undetermined Coefficients. **DO NOT ATTEMPT TO SOLVE THE COEFFICIENTS.**

(b) (3 points) $y'' - 4y' + 8y = 2e^{2t} - 5t^2 + \sin 2t$

(c) (3 points) $y'' - 4y' + 8y = -e^{2t}\sin 2t + 1$

(d) (3 points) $y'' - 4y' + 8y = t^2 e^{-t}\cos 5t$

11. (12 points) Find, **in explicit form**, the solution of the initial value problem

$$y' = \frac{te^{2t}}{2y}, \qquad y(0) = -2.$$

12. (12 points) Given that $y_1(t) = t^3$ is a known solution of the second order linear differential equation

$$t^2y'' - ty' - 3y = 0, \qquad t > 0.$$

Find the general solution of the equation.