

Math 251
October 12, 2005 First Exam

NAME: _____ Section #: _____

There are 9 questions on this exam. Many of them have multiple parts. The point value of each question is indicated either at the beginning of each question or at the beginning of each part where there are multiple part

Show all your work. Partial credit may be given.

The use of calculators, books, or notes is not permitted on this exam.

Please turn off your cell phone before starting this exam.

Time limit 1 hour and 15 minutes.

Question		Score
1	22pt	
2	10pt	
3	10pt	
4	8pt	
5	12pt	
6	10pt	
7	10pt	
8	8pt	
9	10pt	
Total	100pt	

1. a. **2pt** Consider the following differential equation $y' = y + 2t$. Without solving it, determine the slope of the tangent line to the solution at the point $(1, 2)$.

b. **2pt** Find the Wronskian $W(y_1, y_2)$ of the functions $y_1 = \sin t$ and $y_2 = \cos t$

c. **2pt** Suppose y_1 and y_2 are two solutions of the ODE $y'' + (\sin t)y' + y = 0$. and suppose that their Wronskian by $W(y_1, y_2)(t)$ is 2 at $t = 0$. Find $W(y_1, y_2)(t)$ for any t .

For the initial value problems in parts **d.** through **g.** state whether or not one of our two existence and uniqueness theorems for first order ODE's guarantees a unique solution. If the answer is yes and the theorem provides an interval of existence, then state what the interval is **without actually solving the equation.**

d. **2pt** _____ $(t + 2)y' + (y - 1)^{2/3} = 0, \quad y(3) = 0$

e. **2pt** _____ $(t + 2)y' + (y - 1)^{2/3} = 0, \quad y(0) = 1$

f. **2pt** _____ $(t^2 + 2t)y' + y = 0, \quad y(-3) = 0$

g. **2pt** _____ $(t^2 + 2t)y' + y = 0, \quad y(3) = 0$

In parts **h. 2pt** through **j.** assume that $L[y] = y'' + p(t)y' + q(t)y$ and that $p(t)$ and $q(t)$ are continuous functions on the entire real axis $(-\infty, \infty)$.

h. 2pt Circle a pair of functions among the following functions which could be a fundamental set of solutions for the differential equation $L[y] = 0$. (There is more than one correct answer.)

$$y_1 = e^{3+t/2}, \quad y_2 = e^{3+2t}, \quad y_3 = e^{2t-2}, \quad y_4 = 0$$

j. 2pt Suppose that y_1 and y_2 have the following properties: $L[y_1] = t$, and $L[y_2] = 0$. Then one of the following solves the differential equation $L[y] = 2t$. Circle it.

$$y_1/2 + y_2 \quad 2y_1 + 2y_2 \quad 2y_2 \quad y_1 + y_2$$

In parts **k.** through **m.** match the ODE's on the left with a description on the right.

_____ **k. 2pt** $y' = e^{y+t}$

i. linear and separable

ii. linear but not separable

_____ **l. 2pt** $y' = 2 - y$

iii. separable but not linear

_____ **m. 2pt** $y' = t^2 - 3y$

iv. not separable and not linear

2. Consider the autonomous differential equation

$$y' = (y - 6)(2 - y) = -y^2 + 8y - 12$$

a. **2pts** Sketch a direction field for this equation. Indicate the equilibrium solutions in your sketch.

b. **2pts** Which equilibrium solution is (are) **asymptotically stable** and which is (are) **unstable**.

c. **3pts** Find a formula for y'' in terms of y .

e. **3pts** Sketch the graph of the solution with $t \geq 0$ with the initial value $y(0) = 3$ indicating its concavity as accurately as possible.

3. 10 pt Find the general solution to the differential equation

$$ty' = t^6 + 5y \quad t > 0$$

4 a. 6pts Find the general solution to the following IVP

$$3y' = 4y^4t^3$$

b. 2pts Find the solution to the above differential equation that satisfies the initial condition $y(1) = 2$

5. a. **6pt** Find the general solution the following differential equation with initial conditions:

$$y'' + 6y' + 13y = 0$$

- b. **4pt** Solve the following IVP for the above ODE:

$$y(0) = 1, \quad y'(0) = 3$$

- c. **2pt** Solve the following IVP for the above ODE:

$$y(999) = 1, \quad y'(999) = 3$$

6. a. 2pt One of the following differential equations is exact. Circle it:

$$2t + 3y + (2y + 3t)y' = 0$$

$$2t + 3y + (2t + 3y)y' = 0$$

b. 8pt Find the solution to the following differential equation which satisfies $y(1) = 0$:

$$y^2 + t^2 + (e^{2y} + 2ty)y' = 0$$

7. A tank with a capacity of 200 liters initially contains a mixture of 25 grams of salt dissolved in 50 liters of water. A salt water mixture with a concentration of 2 grams/liter enters the tank at the rate of 6 liters/min. Well stirred mixture leaves the tank at 6 liters/min.

a. **4pt** Let $Q(t)$ be the quantity of salt in the tank at time $t \geq 0$. Write down a differential equation and an initial condition for the quantity $Q(t)$ of salt in grams in the tank at any time $t \geq 0$. (**Do not solve it.**)

b. **2pt** Draw a direction field for the ODE.

c. **2pt** Without solving the ODE, determine approximately the quantity of salt in the tank after a long time.

d. **2pt** Without solving the ODE, determine $Q'(0)$.

8. Match the differential equations listed below with the descriptions of long time behavior listed below. (Each description matches only one equation. Please place your answer in the space provided.)

_____ a. 2pt $y'' - y' - 2y = 0$

I. Every solution approaches 0 as $t \rightarrow \infty$

II. Has a nonzero solution that approaches 0 as $t \rightarrow \infty$ and has a nonzero solution that approaches ∞ as $t \rightarrow \infty$

_____ b. 2pt $y'' + 4y' + 4y = 0$

III. Every nonzero solution approaches either ∞ or $-\infty$ as $t \rightarrow \infty$

IV. Every nonzero solution has oscillations which become progressively larger as $t \rightarrow \infty$

_____ c. 2pt $y'' - 4y' + 29y = 0$

V. Every nonzero solution has oscillations which become progressively smaller as $t \rightarrow \infty$

_____ d. 2pt $y'' + y = 0$

VI. Every nonzero solution oscillates with constant amplitude $t \rightarrow \infty$.

9. 10pt The ODE

$$t^2 y'' - ty' + y = 0, \quad t > 0$$

obviously has a solution $y_1 = t$. Use the method of reduction of order to find another solution of this linear homogeneous ODE that is not a constant multiple of y_1 .