

MATH 251
Fall 2004
Exam 1
Oct 12th, 2004

NAME : _____

ID : _____

INSTRUCTOR : _____

There are **12** questions on **10** pages. Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice. For partial credit questions, all work must be shown - **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.
PLEASE DO NOT WRITE IN THE BOX BELOW.

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12: _____
Total: _____

1. (4 points) Identify each of the following equations as linear or non-linear and also determine their order.

(a) $y \frac{dy}{dt} = t$

(b) $\left(\frac{d^2y}{dt^2}\right)^3 + \left(\frac{dy}{dt}\right)^3 + y^3 = t^3$

(c) $\sin(t) \frac{dy}{dt} + t^5 y = (1 - t^2) \frac{d^2y}{dt^2}$

(d) $(1 + y) \sin^2 t + \left(\frac{d^3y}{dt^3} + y\right) \cos^2 t = 1$

2. (6 points) The initial value problem

$$(4 - t^2)y' + \ln(t)y = \sin(t) \quad y(1.33) = 3.14159$$

is certain to have a unique continuous solution on the interval

- (a) $(-4, 4)$
(b) $(-2, 2)$
(c) $(0, 2)$
(d) $(0, 3.14159)$

3. (6 points) The general solution to the equation

$$y'' + 6y' + 9y = 0$$

has the form

- (a) ce^{-3t}
- (b) $c_1e^{-3t} + c_2e^{-3t}$
- (c) $c_1e^{-3t} + c_2te^{-3t}$
- (d) $c_1e^{-3t} + c_2e^{-3t^2}$

4. (6 points) If $y(t)$ is the solution to the initial value problem

$$y' = y(y^2 - 4) \quad y(0) = 3.$$

Then $\lim_{t \rightarrow \infty} y(t) = ?$

- (a) 3
- (b) 2
- (c) -2
- (d) ∞

5. (6 points) Let y_1 and y_2 be two solutions to the linear equation

$$2t^2y'' - ty' - y = 0.$$

Then the Wronskian of y_1 and y_2 must be a constant multiple of

- (a) \sqrt{t}
- (b) t
- (c) 1
- (d) t^2

6. (6 points) Solutions to

$$(x \cos y) \frac{dy}{dx} = \sin y$$

are,

- (a) $y(x) = \tan(x + C)$
- (b) $y(x) = \sin^{-1}\left(-\frac{1}{x} + C\right)$
- (c) $y(x) = \sin^{-1}(Cx)$
- (d) $y(x) = \sin^{-1}\left(\ln(x) - \frac{x}{2} + C\right)$

7. (10 points) Solve the initial value problem.

$$t^3 y' + 4t^2 y = e^{-t} \quad y(1) = 0$$

8. (10 points) A tank has 100gal of water and 100lb of salt mixed in it. Water enters the tank at the rate of 3gal/min with concentration of salt in it, at time t given by e^{-t} lb/gal. A well mixed solution leaves the tank at the same rate of 3gal/min. Find a formula for the amount of salt in the tank at any time t .

Find also the eventual concentration of salt in the tank.

9. (12 points) Show that the equation

$$(4xy - 3)y' + 2(y^2 + x) = 0$$

is exact.

Find the general solution.

10. (10 points) Solve the following initial value problems:

(a)

$$y'' + 2y' + 5y = 0, \quad y(0) = 0, \quad y'(0) = -1.$$

(b)

$$y'' + \frac{1}{t}y' = 1 \quad y(2) = -2\ln(2), \quad y'(2) = 2.$$

11. (12 points) Find α so that the solution to the initial value problem

$$y'' + 3y' - 4y = 0, \quad y(0) = \alpha, \quad y'(0) = 1,$$

converges to 0 as $t \rightarrow \infty$.

12. (12 points) Given that $y_1 = t$ is a solution to the equation

$$t^2 y'' - 4ty' + 4y = 0,$$

use the method of reduction of order to find a second solution which is not a constant multiple of y_1 .

Show that the two solutions above are linearly independent on the interval $(0, \infty)$.