There are 12 questions on 10 pages. Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice. For partial credit questions, all work must be shown - credit will not be given for an answer unsupported by work.

No calculators are allowed.
Please do not write in the box below.
1. (4 points) Identify each of the following equations as linear or non-linear and also determine their order.

(a) \( y \frac{dy}{dt} = t \)

(b) \( \left( \frac{d^2y}{dt^2} \right)^3 + \left( \frac{dy}{dt} \right)^3 + y^3 = t^3 \)

(c) \( \sin(t) \frac{dy}{dt} + t^5 y = (1 - t^2) \frac{d^2y}{dt^2} \)

(d) \( (1 + y) \sin^2 t + \left( \frac{d^3y}{dt^3} + y \right) \cos^2 t = 1 \)

2. (6 points) The initial value problem

\[
(4 - t^2)y' + \ln(t)y = \sin(t) \quad y(1.33) = 3.14159
\]

is certain to have a unique continuous solution on the interval

(a) \((-4, 4)\)

(b) \((-2, 2)\)

(c) \((0, 2)\)

(d) \((0, 3.14159)\)
3. (6 points) The general solution to the equation

\[ y'' + 6y' + 9y = 0 \]

has the form

(a) \( ce^{-3t} \)
(b) \( c_1 e^{-3t} + c_2 e^{-3t} \)
(c) \( c_1 e^{-3t} + c_2 t e^{-3t} \)
(d) \( c_1 e^{-3t} + c_2 e^{-3t^2} \)

4. (6 points) If \( y(t) \) is the solution to the initial value problem

\[ y' = y(y^2 - 4) \quad y(0) = 3. \]

Then \( \lim_{t \to \infty} y(t) = ? \)

(a) 3
(b) 2
(c) −2
(d) \( \infty \)
5. (6 points) Let $y_1$ and $y_2$ be two solutions to the linear equation

$$2t^2 y'' - ty' - y = 0.$$ 

Then the Wronskian of $y_1$ and $y_2$ must be a constant multiple of

(a) $\sqrt{t}$
(b) $t$
(c) 1
(d) $t^2$

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6. (6 points) Solutions to

$$(x \cos y) \frac{dy}{dx} = \sin y$$

are,

(a) $y(x) = \tan(x + C)$
(b) $y(x) = \sin^{-1}\left(-\frac{1}{x} + C\right)$
(c) $y(x) = \sin^{-1}(Cx)$
(d) $y(x) = \sin^{-1}(\ln(x) - \frac{x}{2} + C)$
7. (10 points) Solve the initial value problem.

\[ t^3 y' + 4t^2 y = e^{-t} \quad y(1) = 0 \]
8. (10 points) A tank has 100gal of water and 100lb of salt mixed in it. Water enters the tank at the rate of 3gal/min with concentration of salt in it, at time \( t \) given by \( e^{-t} \)lb/gal. A well mixed solution leaves the tank at the same rate of 3gal/min. Find a formula for the amount of salt in the tank at any time \( t \).

Find also the eventual concentration of salt in the tank.
9. (12 points) Show that the equation

\[(4xy - 3)y' + 2(y^2 + x) = 0\]

is exact.

Find the general solution.
10. (10 points) Solve the following initial value problems:

(a) \[ y'' + 2y' + 5y = 0, \quad y(0) = 0, \quad y'(0) = -1. \]

(b) \[ y'' + \frac{1}{t}y' = 1 \quad y(2) = -2\ln(2), \quad y'(2) = 2. \]
11. (12 points) Find $\alpha$ so that the solution to the initial value problem

$$y'' + 3y' - 4y = 0, \quad y(0) = \alpha, \quad y'(0) = 1,$$

converges to 0 as $t \to \infty$. 
12. (12 points) Given that \( y_1 = t \) is a solution to the equation

\[
t^2 y'' - 4ty' + 4y = 0,
\]

use the method of reduction of order to find a second solution which is not a constant multiple of \( y_1 \).

Show that the two solutions above are linearly independent on the interval \((0, \infty)\).