

Name: _____

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Section: _____

Instructor: _____

EXAM I
MATH 251
October 14, 2003

- This is a closed book exam. No notes or calculators may be used.

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Total: _____

(1) (8 points) For each of the differential equations below, state its order and whether it is linear or nonlinear.

(a) $y' + t^2y = e^t$

(b) $2y'' + 3y' - y = te^{-t}$

(c) $y' = y(y + 1)(y - 1)$

(d) $y''' - 2y' + ty - y^2 = 0$

(2) (5 points) The integrating factor used to solve $2t^2y' + 6ty = e^{-3t}$ is

(a) e^{3t}

(b) e^{3t^2}

(c) t^3

(d) e^{-3t}

- (3) (5 points) The Existence and Uniqueness Theorem guarantees that the solution to

$$\sin(t)y'' + \frac{1}{t-3}y' + e^t y = t^3, \quad y(1) = 0, \quad y'(1) = 1$$

is valid on

- (a) $(0, 3)$
 - (b) $(0, \pi)$
 - (c) $(-\infty, 3)$
 - (d) $(-\infty, \infty)$
- (4) (5 points) Which of the following is the general solution of $y'' + 9y = 0$?
- (a) $c_1 e^{3t} + c_2 e^{-3t}$
 - (b) $c_1 e^{-3t} + c_2 t e^{-3t}$
 - (c) $c_1 e^{-9t} + c_2$
 - (d) $c_1 \cos 3t + c_2 \sin 3t$

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(5) (12 points) Solve the initial value problem

$$y' = \frac{4x^3 - 6x + 3}{2y + 8}, \quad y(1) = -2.$$

Give your answer in explicit form.

(6) (12 points) For the initial value problem

$$(2x + ye^{xy})dx + (xe^{xy} + 1)dy = 0, \quad y(1) = 0$$

- (a) Verify that the equation is exact.
- (b) Solve the initial value problem. You may leave your answer in implicit form.

- (7) (15 points) For the autonomous equation $y' = y^3 + y^2 - 6y$
- (a) Find its equilibrium solutions.
 - (b) For each equilibrium solution, determine its stability. Justify your answer.
 - (c) If $y(0) = 1$, what is $\lim_{t \rightarrow \infty} y(t)$?
 - (d) If $y(0) = -3$, what is $\lim_{t \rightarrow \infty} y(t)$?

- (8) (14 points) A swimming pool is initially filled with $500m^3$ of water with a chlorine concentration of $10\frac{g}{m^3}$. Water containing $50\frac{g}{m^3}$ of chlorine flows into the pool at a rate of $5m^3$ per minute. The well-mixed water in the pool is drained away at the same rate.
- Find the amount of chlorine in the pool at any time $t > 0$.
 - What is the concentration of chlorine in the pool as $t \rightarrow \infty$?

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(9) (12 points) Solve the initial value problem $y'' - 6y' + 9y = 0$, $y(0) = 2$, $y'(0) = 5$.

- (10) (12 points) Find the general solution of $ty'' - 4y' + \frac{4}{t}y = 0$, $t > 0$, given that $y_1 = t$ is a known solution.