MATH 251
Midterm Exam I
Sept. 25, 2001

Name:_____
Student Number:_____
Instructor:_____
Section:_____

There are **9** partial credit questions. **In order to obtain full credit for these problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is in parentheses to the right of the question number.
THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

1:_____	
2:_____	
3:_____	
4:_____	
5:_____	
6:_____	
7:_____	
8:_____	
9:_____	**Do not write in the box to the left**
Total:_____	

1. Give an example of the following:

   (a) (4 points) A first order, nonlinear, autonomous, ordinary differential equation.

   (b) (3 points) A second order, linear, homogeneous, ordinary differential equation.

2. (10 points) Solve explicitly for $y(t)$ in the following initial value problem
   $e^t - yy' = 0$; $y(0) = 1$.

3. (10 points) Find the general solution of the following equation: $(2y - xe^{xy})y' = 2 + ye^{xy}$. You may leave your answer in implicit form.

4. (15 points) A tank is filled with 200 liters of a solution containing 100 grams of salt. A solution containing a concentration of 2 g/liter salt enters the tank at the rate 4 liters/minute and the well-stirred mixture leaves the tank at the same rate. Set up the initial value problem for the amount of salt in the tank at time t, find the particular solution and find the limiting amount of salt in the tank as $t \to \infty$.

5. For the following initial value problem $ty' = 3y + t;\ \ y(4) = -1$

   (a) (5 points) Without solving, find the maximum interval on which we are guaranteed that the problem has a unique solution.

   (b) (10 points) Solve the initial value problem.

6. For the following equation, $\frac{dy}{dt} = (y+1)(y-2)(1-y)$:

   (a) (10 points)determine and classify the critical points (equilibrium solutions).

   (b) (5 points) which direction field represents this equation? Circle your choice and give one reason why your choice is correct.

7. Find the general solution for the following equations:

   (a) (4 points) $y'' - 4y' + 5y = 0$

   (b) (4 points) $y'' + 6y' + 9y = 0$

8. (5 points)Find the particular solution to $y'' - 5y' + 4y = 0, \quad y(0) = 2, \;\; y'(0) = -1.$

(5 points) What is the behavior of the solution as t approaches positive infinity?

9. (10 points) Show $y_1(t) = t^2$ and $y_2(t) = t^3,\ t > 0$ form a fundamental set of solutions for $t^2y'' - 4ty' + 6y = 0$.