

MATH 251  
Midterm Exam I  
Sept. 26, 2000

Name: \_\_\_\_\_  
Student Number: \_\_\_\_\_  
Instructor: \_\_\_\_\_  
Section: \_\_\_\_\_

There are **6** partial credit questions. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.**

**THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.**  
At the end of the examination, the booklet will be collected.

**Time Limit: 75 minutes**

**Do not write in this box.**

1: _____
2: _____
3: _____
4: _____
5: _____
6: _____
Total: _____

1. (16 points) Consider the general  $n$ -th order ordinary differential equation

$$F(t, y(t), y'(t), \dots, y^{(n)}(t)) = 0$$

- (a) What general form must  $F$  have for the equation to be linear?
- (b) Classify the following equations as linear or non-linear, and state their order.
- (i)  $t \frac{d^2 y}{dt^2} + t^2 \frac{dy}{dt} + t^3 y = \cos(t)$
- (ii)  $t \frac{d^3 y}{dt^3} + t^2 \frac{dy}{dt} + t^3 y = \cos(y)$
- (iii)  $\frac{dy}{dx} = \frac{2y-3}{2x+2}$

2. (16 points)

(a) Find the solution to the initial value problem

$$y'' + 3y' + 2y = 0 \quad y(0) = 0 \quad y'(0) = \alpha$$

(b) For what value(s) of  $\alpha$  is the  $\lim_{t \rightarrow \infty} y(t) = 0$ ?

(c) For what value(s) of  $\alpha$  is the  $\lim_{t \rightarrow -\infty} y(t) = 0$ ?

3. (16 points) Consider the autonomous equation

$$y' = y^2 - 3y$$

- (a) Find the equilibrium solutions.
- (b) State the stability of each equilibrium solution. **Justify your answers!**
- (c) If  $y(0) = 0$  what is  $y(1)$ ?

4. (16 points)

(a) solve the initial value problem

$$\sin(t)y' + \cos(t)y = 3t \sin(t) \quad y\left(\frac{\pi}{2}\right) = 0$$

(b) On what interval is the solution guaranteed to exist? **Justify your answers!**

5. (16 points) Solve the following equations. You may leave your answers in implicit form.

(a)  $y' = \frac{x^2}{2y}$

(b)  $x - y + (y - x)y' = 0$

6. (20 points) In preparation for the holiday season, M&M is packaging red and green M&Ms together. The packing machine consists of a very large mixing vat containing 500 gallons of red M&Ms and 500 gallons of green M&Ms. There are two streams, one red the other green, of M&Ms flowing into the tank, each at a rate of 10 gallons per minute. The well mixed M&Ms flow out of the vat into packages at a rate of 20 gallons per minute. A colorblind employee accidentally pours an additional 100 gallons of **blue** M&Ms into the vat.
- (a) Set up an initial value problem that models the situation after the accident.
  - (b) Find an expression for the amount of blue M&Ms in the vat at time  $t$ .
  - (c) How much time must pass before the percentage of blue M&Ms in the tank drops below 1%?