Name: __________

MATH 251 Student Number: __________

Midterm Exam I Instructor: __________

March 1, 2007 Section: __________

This exam has **11** questions for a total of 100 points. There are **2** multiple choice questions. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.**

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION. At the end of the examination, the booklet will be collected.

Do not write in this box.

1: __________
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Total: __________

1. (6 points) A mass weighing 100kg stretches a spring 5m. If there is no damping, which of the following equations describes the motion of the spring?
   (a) $100y'' + \frac{980}{5}y = 0$,
   (b) $100y' + 5y = 0$,
   (c) $100y'' + 5y = \sin 9t$,
   (d) $y'' = \cos(y)$,
   (e) $5y'' + 100y = 0$.

2. (6 points) which equation below describes a system undergoing resonance?
   (a) $y'' + 4y = 0$,
   (b) $y'' + 4y' + 4y = \sin 4t$,
   (c) $y'' + 9y = \sin 9t$,
   (d) $y'' = \cos(y)$,
   (e) $y'' + 9y = 6\cos 3t$.

3. (5 points) Consider the initial value problem

$$(t^2 - 1)y' + (t - 2)y = 4, \qquad y(0) = 5.$$

State the largest inerval in which a unique solution is guaranteed to exist. **Do not solve the equation.**

4. (12 points) For the autonomous equation $y' = y(y-2)(y+4)$

a) Find all equilibrium solutions.

b) Determine which of them are asymptotically stable. Justify your answer.

c) Determine the behavior of solution $y(t)$, which satisfies the initial value $y(1) = 1$, when $t \to +\infty$.

5. (10 points) Consider the initial value problem

$$y + x^3 + (x + 5y)\frac{dy}{dx} = 0, \quad y(0) = 2.$$

a) Verify that the equation is exact.

b) Solve the initial value problem. Leave your answer in implicit form.

6. (15 points) For the equation

$$y'' - y' - 2y = -9e^{-t}.$$

a) Find the general solution of the problem.

b) Find the solution satisfying initial conditions $y(0) = 0$, $y'(0) = 1$.

7. (14 points) Solve each of the following equations. **You may leave your answers in implicit form.**

a)

$$y' = \frac{e^t + 1}{\cos\, y + y} \qquad y(0) = 3.$$

b)

$$t^2 y' + ty = 3, \quad \text{for} \quad t > 0, \quad \text{and} \quad y(1) = 2.$$

8. (6 points) Find the general solution to the following:

$$y'' + 2y' + 3y = 0.$$

9. (10 points) Consider the equation $(t-1)y'' - ty' + y = 0$.

a) Verify that the functions $y_1 = t$ and $y_2 = e^t$ are its solutions.

b) When $t \neq 1$, are y_1 and y_2 linearly independent? Justify your answer.

c) Based on your answers in parts a) and b), and for $t \neq 1$, state the general solution to this equation.

10. (7 points) Consider the second order linear differential equation $ty'' - 2y' + y = 0$. Suppose $y_1(t)$ and $y_2(t)$ are two fundamental solutions of the equation such that $y_1(1) = 2$, $y_1'(1) = 0$, $y_2(1) = 2$, and $y_2'(1) = 2$. Compute their Wronskian $W(y_1(t), y_2(t))$ as a function of t.

11. (9 points) A tank initially contains 100 liters of pure water. A mixture containing a concentration of 5 grams/liter of salt enters the tank at a rate of 2 liters/min. and the well-stirred mixture leaves the tank at the same rate. Let $Q(t)$ be the amount of salt in the tank. Formulate and state an initial value problem satisfied by Q modeling this process. Make sure you write down both an equation and an initial condition that $Q(t)$ must satisfy.

You do not need to solve the equation.