Math 251
February 24, 2005 First Exam

NAME:_____ Section #:_____

There are 9 questions on this exam. Question 9 is worth 12 points. Each other question is worth 11 points. The points assigned to each part of the question are indicated at the start of the part.

Show all your work. Partial credit may be given.

The use of calculators, books, or notes is not permitted on this exam.

Please turn off your cell phone before starting this exam.

Time limit 1 hour and 15 minutes.

Question	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

1. **a. 9pts** Find the general solution to the following ODE

$$ty' + 2y = 2 \qquad\qquad t > 0$$

b. 2pts Find the solution of the above equation which satisfies $y(1) = 0$.

2. **a. 3pts** Verify that the following ODE is exact:

$$2t + e^y + (te^y - \cos y)\frac{dy}{dt} = 0$$

(Show your work.)

b. 6pts Find the general solution to the ODE in Part **a.**
(You may leave your answer in implicit form.)

c. 2pts Find the solution to the ODE in Part **a.** which satisfies $y(1) = \pi/2$.
(You may leave your answer in implicit form.)

3. 11pts Solve the following ODE.

$$y'' = y(y')^3$$

(You may leave your answer in implicit form.)

4. **a. 6pts** A ski slope operator determines that, without producing artificial snow, the volume of snow on the ski slope decreases at a rate equal to $-1/10$ of the volume present. If the operator produces artificial snow at the rate of 10^6 cubic meters per day, then write a differential equation for the volume of snow.
(DO NOT SOLVE the ODE.)

b. 5pts Suppose that after 10 days of operation there are $5\text{x}10^6$ cubic meters of snow on the slopes. Use Euler's method with one step to approximate the amount of snow on 11th day.

5. 11pts Consider the ODE

$$t^2y'' - ty' + y = 0$$

Given that $y_1 = t$ is a solution, find another solution y_2 of this ODE that is not a constant multiple of y_1.

6. **a. 4pts** Find the general solution of $y'' + 6y' + 10y = 0$

b. 3pts Find the general solution of $y'' + 6y' + 9y = 0$

c. 2pts Solve the initial value problem:

$$y'' + 6y' + 9y = 0 \qquad y(0) = 1, \quad y'(0) = 0$$

d. 2pts Find $\lim_{t\to\infty} y(t)$, where $y(t)$ is the solution found in Part **c.**

7. Consider the differential equation

$$y' = -y^2 + 4y - 3$$

a. 3pts Determine all the equilibrium solutions of this ODE.

b. 4pts Sketch a direction field for this ODE.

c. 4pts For each equilibrium solution found in Part **a.** determine whether it is **asymptotically stable** or **unstable.**

8. **a.** **7pts** Let y_1, y_2 be two solutions to the equation $ty" - 2y' - y = 0$. Determine the Wronskian $W(y_1, y_2)$ of y_1 and y_2.

b. **4pts** If $W(y_1, y_2)(2) = 1$, then determine $W(y_1, y_2)(3)$.

9. **a. 3pts** Consider the ODE $(t-1)y' + 3y = 2$. Determine all pairs (t_0, y_0) for which the uniqueness of the solution to the IVP $y(t_0) = y_0$ is **NOT** guaranteed?

b. 3pts Consider the ODE $(t-1)y' + y^{1/3} = 2$. Determine all pairs (t_0, y_0) for which the uniqueness of the solution to the IVP $y(t_0) = y_0$ is **NOT** guaranteed?

c. 3pts Which one of the following is a second order ODE? Circle it.

$$y^2 = \cos^2 t \qquad y(y')^2 = \sin t \qquad \frac{y''}{y} = \tan t$$

d. 3pts Which one of the following is a linear ODE? Circle it.

$$t^4 y' + t^3 y = \sin^2 t \qquad 1 = y\, y' \qquad t = y' e^y$$