Math 251H Second Midterm Exam

75 minutes

March 23, 2009

SAMPLE EXAM.

1. A 3 kg mass is suspended from a spring, which stretches the spring 5 m from its natural length. The system is placed into the liquid with damping constant 14 Newton-seconds per meter. At t = 0, the system is at rest at its equilibrium position, then receives an external force of $6\cos(\omega t)$ Netwons. Assume ω is positive and $g = 10m/s^2$.

(a) Set up an initial value problem that describes the motion of the mass. Be sure to explain any variables that appear in your equation.

(b) For which value of ω will the system have resonance?

(c) Find the steady-state solution.

2.

(a) Find the Laplace transform of

$$f(t) = \begin{cases} 9, & 0 \le t < 3, \\ t^2, & t \ge 3. \end{cases}$$

(b) Find the Laplace transform of

$$f(t) = e^t \sin(\sqrt{2}\, t).$$

(c) Find the inverse Laplace transform of

$$F(s) = \frac{s-2}{s^2 + 2s + 10}.$$

3. Solve the following initial value problem using Laplace transform,

$$y^{(4)} - 16y = 0, y(0) = 1, y'(0) = 2, y''(0) = 0, y^{(3)}(0) = 0.$$

4. Solve the following initial value problem,

$$y'' + 6y' + 13y = g(t),$$

where

$$g(t) = \begin{cases} t - 1, & 0 \le t < 1, \\ 0, & t \ge 1. \end{cases}$$

sketch the solution in time.

5. Solve the following initial value problem.

$$y'' - 2y = 4e^{-2t}\delta(t-1), y(0) = 0, y'(0) = 1.$$

Sketch the solution.