

MATH 250
First Midterm Exam
Feb 26, 2003

Name: _____
Student Number: _____
Instructor: _____
Section: _____

There are **8** multiple choice questions and **6** partial credit questions. **In order to obtain full credit for the partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work on a partial credit problem. THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.**

For multiple choice problems, write the letter of your choice in the space provided below.

Your Answer :

Points awarded

1. (5 pts) _____

Q. 9 (10 pts) _____

2. (5 pts) _____

Q. 10 (10 pts) _____

3. (5 pts) _____

Q. 11 (10 pts) _____

4. (5 pts) _____

Q. 12 (10 pts) _____

5. (5 pts) _____

Q. 13 (10 pts) _____

6. (5 pts) _____

Q. 14 (10 pts) _____

7. (5 pts) _____

8. (5 pts) _____

1. (5 points) Which figure represents the solution of the differential equation

$$2u'' + u' + \frac{17}{8}u = 0, \quad u(0) = 2, \quad u'(0) = 0 \quad ?$$

- $-2\exp(-t/4)\cos(t) + 1/2\exp(-t/4)\sin(t)$
 - $2\exp(-t/4)\cos(t) + 1/2\exp(-t/4)\sin(t)$
 - $2\exp(-t/4)$
 - $(t+2)\cos(t)$
2. (5 points) Knowing the differential equation

$$4y'' + 3y' + y = 0$$

for a spring-mass system, which of the following statement is **not** true?

- The equation describes a damped spring-mass system with a mass 4 and spring constant 1.
 - The system has a quasi-period of $\frac{16\pi}{3}$
 - Irrespective of the initial conditions, the system will eventually come to rest.
 - The quasi-frequency of the system is $\frac{\sqrt{7}}{8}$
3. (5 points) If the Laplace transform of f is

$$F(s) = \frac{s-1}{s^2-s-2}$$

and $f(0) = -1$, $f'(0) = 3$, then the Laplace transform of f'' is

- $\frac{s-1}{s^2-s-2} - s - 3$
 - $\frac{s^3-s^2}{s^2-s-2} + s - 3$
 - $\frac{s^2-s}{s^2-s-2} - s - 3$
 - $\frac{s^3-s^2}{s^2-s-2} + s + 3$
4. (5 points) The inverse Laplace transform of $F(s) = \frac{s-1}{s^2+3s+2}$ is
- $-2e^{-t} + 3e^{-2t}$
 - $-e^{-2t} + 3e^t$
 - $2e^{-t} - 3e^{-2t}$
 - $-2e^{-2t} + 3e^{-t}$

5. (5 points) What is the smallest time T for which the solution

$$y(t) = e^{-t}[\cos(2t) - \frac{1}{2}\sin(2t)]$$

satisfies $|y(t)| < 1/2$ for all $t > T$?

- $\ln 5$
 - $\frac{1}{2}\ln 5$
 - $\ln 2$
 - there is no such a T
6. (5 points) Which of the following is the solution of the IVP

$$9y'' + 6y' + y = 0, \quad y(1) = 0, \quad y'(1) = 1?$$

- $y(t) = e^{1/3}e^{-t/3} - e^{1/3}te^{-t/3}$
 - $y(t) = te^{-t/3}$
 - $y(t) = t^3/3 - 1/3$
 - $y(t) = (t - 1)e^{-(t-1)/3}$
7. (5 points) Using the method of undetermined coefficients, a **particular** solution of

$$y'' + y = e^{3t} - 4t \cos 5t$$

has the form:

- $Ae^{3t} + (Bt + C) \cos 5t$
 - $Ae^{3t} + Bt \cos 5t + Ct \sin 5t$
 - $Ae^{3t} + (Bt + C) \cos 5t + (Dt + E) \sin 5t$
 - $Ae^{3t} + Bt \cos 5t$
8. (5 points) Which of the following improper integrals defines the Laplace transform of

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < 4 \\ e^{2t} & \text{if } 4 \leq t \end{cases} \quad ?$$

- $\int_0^\infty e^{-(s+2)t} dt$
- $\int_4^\infty e^{-(s-2)t} dt$
- $\int_0^\infty e^{-(s-2)t} dt$
- $\int_4^\infty e^{-(s+2)t} dt$

9. (10 points) A spring-mass system moving on a flat table with a friction force of $-2|u'|$ Newtons. It takes 1 Newton force to stretch the spring 10 cm from its equilibrium position. If an external force of $3\cos(t)$ is applied to the system, do the following:

Write down the differential equation that describes the motion of the mass.

What is the general solution of the differential equation?

What is the amplitude of the steady-state solution?

10. (10 points) Find the solution of the following initial value problem using the method of undetermined coefficients:

$$y'' + 3y' + 2y = 2e^t, \quad y(0) = 0, \quad y'(0) = 2$$

11. (10 points) Use the method of reduction of order to find a second solution of the differential equation:

$$t^2y'' + 2ty' - 6y = 0, \quad t > 0,$$

knowing that $y_1(t) = t^2$ is a solution. What is the **general** solution of the above equation? (NO credit will be given to unjustified answers.)

12. (10 points) Find the solution of the following initial value problem using the method of undetermined coefficients:

$$y'' - 4y' + 3y = te^{3t}, \quad y(0) = 1, \quad y'(0) = 0.$$

13. (10 points) Find the general solution of the following problem

$$y'' - 4y' + 5y = 0.$$

14. (10 points) Solve the following initial value problem

$$y'' + 10y' + 25y = 0, y(0) = 1, y'(0) = 0.$$