

MATH 250
Midterm Exam I
September 29, 2009

Name: _____
Student Number: _____
Instructor: _____
Section: _____

This exam has 14 questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.**

YOU MAY NOT USE A CALCULATOR ON THIS EXAM. PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE. At the end of the examination, the booklet will be collected.

Do not write in this box.

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Total: _____

1. (12 points) For parts (a) through (d) below, a list of differential equations is given. For each part, write down the letter corresponding to the equation on the list with the specified properties. There is only one correct answer to each part.

A. $y'' + 4y' - y = t^3 + e^{-2t}$

B. $y' + 4y = t^2 \sin(3t)$

C. $y' - y = 5$

D. $y''' = y - 4t^2$

E. $y'' = y' - 10y$

F. $y' = y^3 + 3y^2 + 3y + 1$

G. $y''' = 27 - y^3$

(a) (3 points) First order nonlinear equation

(b) (3 points) First order linear, autonomous, equation

(c) (3 points) Second order nonhomogeneous linear equation

(d) (3 points) Third order linear equation

2. (5 points) Consider the two statements below. Which is true?

(I) A first order equation could be both linear and separable.

(II) $y' = e^{-10y} + 1$ is an example of an autonomous equation that has no equilibrium solution.

(a) I

(b) II

(c) Both I and II are true.

(d) Neither is true.

3. (5 points) Find the general solution of

$$y' - 6y = te^{6t}.$$

(a) $y = \frac{1}{12}te^{12t} + Ce^{6t}$

(b) $y = e^{-6t} + C$

(c) $y = te^{-6t} + Ce^{-6t}$

(d) $y = \frac{1}{2}t^2e^{6t} + Ce^{6t}$

4. (5 points) Consider the initial value problem

$$(t^2 - 4)y' - \frac{\cos \pi t}{t} y = \frac{4}{t-6}, \quad y(1) = 7.$$

Without solving the equation, what is the largest interval in which a unique solution is guaranteed to exist?

- (a) $(0, 2)$
 - (b) $(0, 6)$
 - (c) $(6, \infty)$
 - (d) $(2, 6)$
5. (5 points) Find the solution of the initial value problem

$$y' = \frac{6x^2 + 5}{y + 1}, \quad y(1) = -2.$$

- (a) $y = -1 + \sqrt{4x^3 + 10x - 13}$
- (b) $y = -1 - \sqrt{4x^3 + 10x - 13}$
- (c) $y = -1 + \sqrt{2x^3 + 5x - 6}$
- (d) $y = -1 - \sqrt{2x^3 + 5x - 6}$

6. (5 points) Consider all the solutions of the equation

$$y' + y = 10.$$

As $t \rightarrow \infty$, they will

- (a) all approach $-\infty$.
- (b) all approach 0.
- (c) all approach 10.
- (d) some approach $+\infty$, some approach $-\infty$.

7. (5 points) Which pair of functions below cannot be a fundamental set of solutions?

- (a) e^{5t} , $2e^{-5t}$
- (b) $\cos t$, $-9 \sin t$
- (c) $2e^{-t}$, $4e^{-t+3}$
- (d) e^{2t} , $e^{2t} - 6$

8. (5 points) Suppose $y_1(t) = t$ and $y_2(t) = t^2$ are both solutions of the second order linear equation

$$y'' + p(t)y' + q(t)y = 0.$$

All of the functions below are also solutions of the same equation, EXCEPT

- (a) $y = t^2 - 1$
 - (b) $y = 5t^2$
 - (c) $y = -9t^2 + 17t$
 - (d) $y = 0$
9. (5 points) Which of the following second order homogeneous linear equations below does not have $y = e^{-4t}$ as a solution?

- (a) $2y'' + 8y' = 0$
- (b) $y'' - 2y' - 8y = 0$
- (c) $y'' - 16y = 0$
- (d) $-y'' - 5y' - 4y = 0$

10. (5 points) Let $y_1(t)$ and $y_2(t)$ be any two solutions of the second order linear equation

$$(t - 5)y'' + 6y' + \sec^2(t)y = 0.$$

What is the general form of their Wronskian, $W(y_1, y_2)(t)$?

- (a) Ce^{6t}
- (b) Ce^{-6t}
- (c) $C(t - 5)^6$
- (d) $\frac{C}{(t - 5)^6}$

11. (5 points) Find the general solution of

$$y'' + 2y' + 17y = 0.$$

- (a) $y = C_1 e^t \cos(4t) + C_2 e^t \sin(4t)$
- (b) $y = C_1 e^{-t} \cos(4t) + C_2 e^{-t} \sin(4t)$
- (c) $y = C_1 e^t \cos(8t) + C_2 e^t \sin(8t)$
- (d) $y = C_1 e^{-t} \cos(8t) + C_2 e^{-t} \sin(8t)$

12. (5 points) Let $y(t)$ be the solution of the initial value problem

$$y'' + y' - 6y = 0, \quad y(0) = 7, \quad y'(0) = \lambda.$$

Suppose $\lim_{t \rightarrow \infty} y(t) = 0$, find the value of λ .

- (a) 14
- (b) 0
- (c) -7
- (d) -21

13. (18 points) Consider the first order autonomous equation

$$y' = y^2(9 - y^2).$$

Answer the following questions without solving the equation.

(a) (3 points) Find all of its equilibrium solutions.

(b) (6 points) For each equilibrium solution, classify its stability. Justify your answer.

(c) (3 points) If $y(10) = -3$, what is $y(0)$?

(d) (3 points) If $y(1) = -\pi$, what is $\lim_{t \rightarrow \infty} y(t)$?

(e) (3 points) If $y(-\pi) = 1$, what is $\lim_{t \rightarrow \infty} y(t)$?

14. (15 points) A mixing tank initially contains 100 liters of fresh water. Solution containing 10 grams/liter of carbon dioxide flows into the tank at the rate of 2 liters/min. The well-stirred mixture flows out of the tank at the same rate.

(a) (6 points) Set up an initial value problem (give both a differential equation and an initial condition) modeling this process.

(b) (6 points) Solve the initial value problem thus obtained.

(c) (3 points) What is the concentration of carbon dioxide in the tank as $t \rightarrow \infty$?