1. Consider the function

$$f(x) = \begin{cases} 2-x & x < -1 \\ x & -1 \le x < 1 \\ (x-1)^2 & x \ge 1 \end{cases}$$

and determine the values of $a$ for which $\lim_{x \to a} f(x)$ exists.

a) $(-\infty, 1) \cup (1, \infty)$

b) $(-\infty, -1) \cup (-1, \infty)$

c) $(-\infty, \infty)$

d) $(-\infty, -1) \cup (1, \infty)$

e) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

2. If $f(x)$ and $g(x)$ are continuous functions with $f(3) = 5$ and $\lim_{x \to 3}[6f(x) - g(x)] = 4$, find $g(3)$.

a) 26

b) 34

c) 5

d) 6

e) 0

3. A curve has the equation $y = h(x)$. Choose an expression for the slope of the secant line through the points $P(5, h(5))$ and $Q(x, h(x))$.

a) $\dfrac{h(x) - h(5)}{x - 5}$

b) $\dfrac{h(5) - h(x)}{x - 5}$

c) $\dfrac{h(x) - h(5)}{5 - x}$

d) $\dfrac{x - 5}{h(x) - h(5)}$

e) $\dfrac{h(x) - x}{h(5) - 5}$

4. Which of the following equals the derivative of the function $f(x) = 4\sqrt{x}$?

a) $\lim_{h \to 0} \dfrac{4\sqrt{x} - 4\sqrt{h}}{h}$

b) $\lim_{h \to 0} \dfrac{4\sqrt{x+h} - 4\sqrt{x}}{h}$

c) $\lim_{h \to 0} \dfrac{4\sqrt{x+h} - 4\sqrt{h}}{h}$

d) $\lim_{h \to 0} 4\sqrt{x+h} - 4\sqrt{h}$

e) $\lim_{h \to 0} \dfrac{4\sqrt{x} + h}{h}$

5. If $h(-4) = 0$ and $h'(-4) = 2$, find $\left(\dfrac{h(x)}{x}\right)'$ at $x = -4$.

a) $-\dfrac{1}{2}$

b) $-1$

c) $-\dfrac{6}{7}$

d) $-\dfrac{3}{11}$

e) $\dfrac{9}{14}$

6. Find an equation of the tangent line to the curve $y = 9\tan x$ at the point $\left(\dfrac{\pi}{4}, 9\right)$.

a) $y = 26x + 19\left(1 - \dfrac{\pi}{2}\right)$

b) $y = 27x + 7\left(1 - \dfrac{\pi}{2}\right)$

c) $y = 25x + 16\left(9 - \dfrac{\pi}{2}\right)$

d) $y = 12x + 17\left(1 - \dfrac{\pi}{4}\right)$

e) $y = 18x + 9\left(1 - \dfrac{\pi}{2}\right)$

7. Find the derivative of the function $y(x) = -4\sec 4x$.

a) $-16\sec 4x \tan 4x$

b) $-15\sec 4x \sin 4x$

c) $-16\sec 4x \cos 4x$

d) $-4\sec 4x \tan 4x$

e) $16\tan^2 4x$

8. Find $y'$ by implicit differentiation for $12x^2 + 4y^2 = 9$.

a) $y' = -3xy$

b) $y' = \dfrac{-3x}{y}$

c) $y' = \dfrac{3x^2}{y^2}$

d) $y' = \dfrac{-48x}{y}$

e) $y' = \dfrac{x}{y}$

9. Find the second derivative of the function $y = \dfrac{x}{5-x}$.

   a) $\dfrac{-10}{(5-x)^2}$

   b) $\dfrac{-1}{(5-x)^2}$

   c) $\dfrac{-5}{(5-x)^3}$

   d) $\dfrac{5}{(5-x)^2}$

   e) $\dfrac{10}{(5-x)^3}$

10. Find the absolute minimum value of $y = 10x^2 + \dfrac{20}{x}$ on the interval $[0, 20]$.

   a) 10

   b) 30

   c) 40

   d) 20

   e) 0

11. Find the absolute maximum of the function $f(x) = \sin(10x) + \cos(10x)$ on the interval $\left[0, \dfrac{\pi}{30}\right]$.

   a) $\sqrt{3}$

   b) $\sqrt{2}$

   c) 2

   d) 1

   e) 0

12. Find the critical numbers of $f(x) = x^4(x-5)^3$.

   a) $0, 5, \dfrac{20}{13}$

   b) $0, 4, \dfrac{20}{13}$

   c) $0, 4, \dfrac{20}{7}$

   d) $0, 5, \dfrac{20}{7}$

   e) $3, 4, 5$

13. Find $\lim\limits_{x\to\infty} f(x)$ if $\dfrac{5x-2}{x} < f(x) < \dfrac{5x^2-4x+14}{x^2}$ for $x > 10$.

   a) 5

   b) 9

   c) 19

   d) 2

   e) 10

14. Find $f(x)$ if $f''(x) = 12x + 24x^2$.

   a) $f(x) = 6x^3 + 4x^4 + Cx + D$

   b) $f(x) = 4x^3 + 8x^4 + Cx + D$

   c) $f(x) = 2x^3 + 2x^4 + Cx + D$

   d) $f(x) = 12x^2 + 24x^3 + C$

   e) $f(x) = 6x^2 + 8x^3 + Cx + D$

15. Find the volume generated by rotating the region bounded by the curves $y = \dfrac{1}{x}, y = 0, x = 1$ and $x = 3$ about the $y-$axis.

   a) $\pi$

   b) $5\pi$

   c) $2\pi$

   d) $4\pi$

   e) $3\pi$

16. If $g(x) = \displaystyle\int_x^6 5\tan(t)dt$, find $g'(x)$.

   a) $5\tan(x)$

   b) $-5\tan(x)$

   c) $5\tan(6)$

   d) $5\sec^2(t)$

   e) $5\sec^2(x)$

17. Evaluate the indefinite integral $\displaystyle\int \sec^5 x \tan x dx$.

   a) $-\dfrac{1}{5}\sec^5 x + C$

   b) $-\dfrac{1}{6}\csc^6 x + C$

   c) $\dfrac{1}{5}\sec^5 x + C$

   d) $\dfrac{1}{5}\tan^5 x + C$

   e) $\dfrac{1}{6}\sec^6 x + C$

18. Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the $x-$axis.

a) $\frac{5}{12}$

b) $\frac{1}{5}$

c) $\frac{1}{3}$

d) $\frac{1}{12}$

e) $\frac{17}{12}$

19. Find the volume of the solid obtained by rotating the region bounded by $x = y^2$ and $x = 3y$ about the $y-$axis.

a) $-81\pi$

b) $\frac{162\pi}{5}$

c) $\frac{81\pi}{5}$

d) $486\pi$

e) $3\pi$

20. Find an expression as a limit for the area from $x = 4$ to $x = 7$ under the curve $y = x^2$.

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 + \frac{4i}{n}\right)^2 \left(\frac{4}{n}\right)$

b) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 + \frac{5i}{n}\right)^2 \left(\frac{2}{n}\right)$

c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 + \frac{6i}{n}\right)^2 \left(\frac{4}{n}\right)$

d) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 + \frac{5i}{n}\right)^2 \left(\frac{5}{n}\right)$

e) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 + \frac{3i}{n}\right)^2 \left(\frac{3}{n}\right)$

21. (10pts.) A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point $(4, 2)$, its $x-$coordinate increases at a rate of 3cm/sec. How fast is the square of the distance from the origin to the particle changing at this instant?

22. (10pts.) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

23. (10pts.) A region $R$ in the $xy-$plane is bounded by $y = \sqrt{x}$ and $y = x$.

a) (2 pts.) Sketch the region $R$. Be sure to label the curves and intersection points.

b) (4 pts.) Set up the integral (by using the **Washer Method**) which measures the volume of the solid generated by revolving $R$ around the axis $x = 2$. DO NOT evaluate the integral.

c) (4 pts.) Set up the integral (by using the **Cylindrical Shell Method**) which measures the volume of the solid generated by revolving $R$ around the axis $x = 2$. DO NOT evaluate the integral.

24. (20 pts.) Consider the function $f(x) = \frac{1}{1-x^2}$ where $f'(x) = \frac{2x}{(1-x^2)^2}$ and $f''(x) = \frac{6x^2+2}{(1-x^2)^3}$.

a) (2 pts.) Find the domain of $f(x)$.

b) (1 pt.) Find the $x$- and $y$-intercepts of the graph of $f(x)$.

c) (1 pt.) Is the graph of $f(x)$ symmetric about the $y$-axis, symmetric about the origin or neither?

d) (3 pts.) Find the horizontal, vertical and slant asymptotes for $f(x)$ if any exist.

e) (3 pts.) Find all the critical numbers for $f(x)$.

f) (2 pts.) Find the interval(s) where $f(x)$ is increasing and the interval(s) where $f(x)$ is decreasing.

g) (2 pts.) Find the local minimum and local maximum points of $f(x)$.

h) (2 pts.) Find the interval(s) where $f(x)$ is concave up and the interval(s) where $f(x)$ is concave down.

i) (2 pts.) Find all the inflection points for $f(x)$.

j) (2 pts.) Sketch the graph of $f(x)$.