MATH 140 FINAL EXAMINATION Spring 2003

Name_____ ID #_____ Section #_____

There are 14 multiple choice, 1 true/false and 4 partial credit type questions.

To obtain credit for all partial credit problems all work must be shown.

The examination is worth 150 points. The point value of each problem is shown next to the problem number.

At the end of the examination, the booklet will be collected.

THE USE OF CALCULATORS, BOOKS, NOTES, ETC., IS NOT PERMITTED IN THIS EXAMINATION.

THERE ARE 19 PROBLEMS ON 14 PAGES, INCLUDING THIS ONE.
CHECK YOUR BOOKLET NOW.

The box below is for the instructor's use.

MC ..... (84)

(T/F) ..... (10)

16 ..... (13)

17 ..... (13)

18 ..... (12)

19 ..... (18)

Total ..... (150)

6 pts 1. Find the derivative of $f(x) = \dfrac{\sin x}{2 + \cos x}$.

a) $-\left(\dfrac{2\cos x + 1}{2 + \cos x}\right)$

b) $\dfrac{\cos x}{\sin x}$

c) $-\dfrac{(2\cos x + 1)}{(2 + \cos x)^2}$

d) $\dfrac{\cos x}{2x + \sin x}$

e) $\dfrac{2\cos x + 1}{(2 + \cos x)^2}$

6 pts 2. Which of the following functions is an antiderivative of $f(x) = 3x^2 - 1$?

a) $F(x) = 6x$

b) $F(x) = x^3 - 1$

c) $F(x) = x^3 - x + \pi$

d) $F(x) = 6x - x$

e) None of the above

6 pts 3. Find the equation of the line tangent to the graph of $f(x) = 1 + \sin x$ at $x = \pi/6$.

a) $y = 3/2 + \dfrac{\sqrt{3}}{2}(x - \pi/6)$

b) $y = 3/2 + \cos x(x - \pi/6)$

c) $y = 1/2 + \dfrac{\sqrt{3}}{2}(x - \pi/6)$

d) $y = \dfrac{\sqrt{3}}{2}(x - \pi/6)$

e) $y = 1 + \cos x(x - \pi/6)$

6 pts 4. Evaluate $\lim\limits_{x \to 1} \dfrac{\sqrt{3x - 2} - 1}{x - 1}$.

a) 1/2

b) 3/2

c) 1

d) 3

e) 2/3

6 pts 5. Solve the initial value problem $\dfrac{dy}{dx} = \sin 2x$, $y(0) = \dfrac{1}{2}$.

a) $y = \cos 2x - \dfrac{1}{2}$

b) $y = -\cos 2x + \dfrac{3}{2}$

c) $y = -2\cos 2x + \dfrac{5}{2}$

d) $y = -\dfrac{1}{2}\cos 2x + \dfrac{1}{2}$

e) $y = -\dfrac{1}{2}\cos 2x + 1$

6 pts 6. Compute $\displaystyle\int \frac{\sec^2(\sqrt{x}-4)}{\sqrt{x}}dx$.

a) $1/2\tan(\sqrt{x}-4) + C$

b) $2\tan(\sqrt{x}-4) + C$

c) $\tan(\sqrt{x}-4) + C$

d) $2\tan(\sqrt{x}-4)$

e) $1/3\big(\sec(\sqrt{x}-4)\tan(\sqrt{x}-4)\big)^3 + C$

6 pts 7. Find the graph which best approximates $f(x)$ if the graph of $f'(x)$ is:

6 pts 8. Evaluate $\lim_{x \to 1} \dfrac{x^2 + 2x - 3}{x^2 - 3x + 2}$.

a) -4

b) 0

c) 1

d) ∞

e) Does not exist.

6 pts 9. Find the area between the graphs of $y = x^2 - 1$ and $y = -x^2 + 1$ on the interval $[-1, 1]$.

a) $4/3$

b) $8/3$

c) 0

d) 1

e) $-4/3$

6 pts 10. If $F(x) = \int_2^{\sqrt{x}} \sin t^2 dt$ what is $F'(x)$?

a) $F'(x) = \dfrac{\sin x}{2\sqrt{x}}$

b) $F'(x) = \sin x$

c) $F'(x) = \sin x + C$

d) $F'(x) = \sin x - \sin(4)$

e) $F'(x) = -\cos x$

6 pts 11. If $f(x) = 2x^3 - 18x$, find the x-coordinates of all local extrema.

a) Local max at $x = 3$; Local min at $x = -3$.

b) No local max; Local min at $x = -\sqrt{3}$.

c) Local max at $x = \sqrt{3}$; Local min at $x = -\sqrt{3}$.

d) Local max at $x = -\sqrt{3}$; Local min at $x = \sqrt{3}$.

e) f has no local extrema.

6 pts 12. Find the derivative of $f(x) = \cos^2(3x^3 + 2)$.

a) $12\cos(3x^3 + 2)\sin(3x^3 + 2)$

b) $-6x^2\cos(3x^3 + 2)\sin(3x^3 + 2)$

c) $-2\cos(3x^3 + 2)\sin(3x^3 + 2)$

d) $-18x^2\cos(3x^3 + 2)\sin(3x^3 + 2)$

e) $\cos^2(9x^2 + 2)$

6 pts 13. What is the length of the curve $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 1$?

a) $\frac{2}{3}(2^{3/2} - 1)$

b) $\frac{3}{2}(2^{3/2} - 1)$

c) $\frac{2}{3}(1 - 2^{3/2})$

d) $\frac{3}{2}(1 - 2^{3/2})$

e) $2^{3/2}(\frac{3}{2} - 1)$

6 pts 14. Let R be the region bounded by the curves $y = x$ and $y = x^2$ on the interval $[0, 1]$. Which of the following integrals is equal to the volume of the solid of revolution obtained by revolving R about the x-axis?

a) $\pi \int_0^1 (x - x^2)^2 dx$

b) $\pi \int_0^1 x^2 - x^4 dx$

c) $\pi \int_0^1 x^4 - x^2 dx$

d) $\pi \int_0^1 (x^2 - x)^2 dx$

e) $2\pi \int_0^1 (x - x^2) dx$

10 pts 15. For each question below circle true or false. This is *not* a partial credit type problem – i.e. you do not have to give reasons for your answers. Let f be a continuous function on some interval $[a, b]$ and let $c \in (a, b)$ be any point in the interval.

a) (2 pts) If $f'(c) = 0$ then f has a local extremum at c. (TRUE/FALSE)

b) (2 pts) If $f''(c) = 0$ then f has an inflection point at c. (TRUE/FALSE)

c) (2 pts) f' is continuous. (TRUE/FALSE)

d) (2 pts) If $F(x) = \int_a^x f(t)dt$ then F' is continuous. (TRUE/FALSE)

e) (2 pts) If $F(x) = \int_a^x f(t)dt$ then F is continuous. (TRUE/FALSE)

13 pts 16. Recall that the graph of $x^2 + y^2 = r^2$ is the circle, centered at the origin, of radius r.

a) (3 pts) Sketch the graph of $y = \sqrt{r^2 - x^2}$.

b) (5 pts) Write down an integral which expresses the volume of the solid of revolution obtained by rotating the graph of $y = \sqrt{r^2 - x^2}$ about the x-axis.

c) (5 pts) By computing the integral you set up in part b), show that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

13 pts 17. Assume f is differentiable on $[a, b]$, $x_0 \in (a, b)$ and $f'(x_0) \neq 0$. Recall that the graph of f passes through the point $(x_0, f(x_0))$.

a) (5 pts) Write down the equation of the line tangent to the graph of f at the point x_0.

b) (3 pts) Find the point where the line you found in part a) crosses the x-axis.

c) (5 pts) If x_0 is the initial guess in Newton's Method for $f(x) = 0$, what is x_1 (i.e. what is the next number generated by Newton's Method)?

12 pts 18. The highway department is planning to build a picnic area along a major highway. It is to be a rectangle with fence along all four sides and with an area of 1200 square meters. The cost of the fence for the two sides parallel to the highway is \$6 per meter while the cost of the fence perpendicular to the highway is \$2 per meter.

a) (10 pts) Find the dimensions of the fence which minimizes the cost. (You *must* use the *second derivative test* to prove that the cost is minimum!)

b) (2 pts) What is the total cost of building the fence?

18 pts 19. A 15 foot ladder is leaning against a house when its base starts to slide away. By the time the base is 9 feet from the house the base is moving at a rate of 4 ft/sec.

a) (6 pts) How fast is the top of the ladder sliding down then?

b) (6 pts) At what rate is the angle formed by *the house* and the ladder changing then?

c) (6 pts) At what rate is the area of the triangle formed by the ladder, wall and the ground changing then?