

MATH 140

NAME _____

FINAL EXAM

STUDENT NUMBER _____

DECEMBER 18, 2003

INSTRUCTOR _____

FORM A

SECTION NUMBER _____

This examination will be machine processed by the University Testing Service. Use only a number 2 pencil on your scantron. On your scantron identify your name, this course (Math 140) and the date. Code and blacken the corresponding circles on your scantron for your student I.D. number, section number and your **test form**.

There are 18 multiple choice questions worth a total of 108 points. For each problem **five** possible answers are given, only one of which is correct. **Circle** the correct answer in your exam booklet **and blacken** the corresponding space on the **scantron form**. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle). Check frequently to be sure the problem number on the test is the same as the problem number of the scantron. There are **3** partial credit questions worth a total of 42 points. **In order to obtain full credit for these problems, all work must be shown. Credit will not be given for an answer not supported by work.** The point value for each question is shown to the left of the question number.

ALL CALCULATORS, NOTES, BOOKS, ETC. ARE FORBIDDEN.

MC (108 pts.) _____
19. (10 pts.) _____
20. (16 pts.) _____
21. (16 pts.) _____
Total _____

**Do not
write in
the box to
the left.**

- 6 pts 1. Find $\lim_{t \rightarrow 3} \frac{\sqrt{t+1} - 2}{2t - 6}$.
- a) $\frac{1}{2}$
 - b) $\frac{1}{4}$
 - c) $\frac{1}{8}$
 - d) 0
 - e) The limit does not exist.

- 6 pts 2. Find $\frac{d}{dx} \left(\frac{x}{\sqrt{1-x^2}} \right)$.
- a) $\frac{\sqrt{1-x^2} + x^2}{(1-x^2)^{3/2}}$
 - b) $\frac{1 + x\sqrt{1-x^2}}{2(1-x^2)^{3/2}}$
 - c) $\frac{2-x-2x^2}{2(1-x^2)^{3/2}}$
 - d) $\frac{\sqrt{1-x^2} - x}{2(1-x^2)^{3/2}}$
 - e) $\frac{1}{(1-x^2)^{3/2}}$

6 pts 3. If $f(\theta) = \theta \sin \theta$, find $f''\left(\frac{\pi}{2}\right)$.

- a) $\frac{\pi}{2}$
- b) 1
- c) 0
- d) -1
- e) $-\frac{\pi}{2}$

6 pts 4. Find the slope of the tangent line to the curve defined by $x^2 + 3xy + y^2 - x = 10$ at the point $(1, 2)$.

- a) -1
- b) $\frac{3}{7}$
- c) 0
- d) $\frac{10}{7}$
- e) 14

6 pts 5. Consider the function $f(x) = \frac{\sin x}{x}$. Which of the following statements is true?

- a) f has a vertical asymptote at $x = 0$.
- b) f has a removable discontinuity at $x = 0$.
- c) f has a jump discontinuity at $x = 0$.
- d) f is continuous but not differentiable at $x = 0$.
- e) f is continuous and differentiable at $x = 0$.

6 pts 6. Find

$$\lim_{x \rightarrow 4^-} \frac{x^2 - 5x + 4}{|x - 4|}$$

- a) 5
- b) 3
- c) -3
- d) -5
- e) The limit does not exist.

6 pts 7. Two men start walking from the same point. One travels south at 2 mi/h and the other travels east at $\frac{3}{2}$ mi/h. At what rate is the distance between the two men increasing four hours later?

a) $\frac{7}{2}$ mi/h

b) $\frac{7}{4}$ mi/h

c) $\frac{1}{10}$ mi/h

d) $\frac{5}{2}$ mi/h

e) $\frac{1}{2}$ mi/h

6 pts 8. Find the x -coordinates of all local maxima of the function $f(x) = \frac{x}{x^2 + 9}$.

a) $x = 3$ only

b) $x = -3$ only

c) $x = 3$ and $x = -3$

d) $x = -9$ only

e) none of the above

6 pts 9. Determine the open interval(s) on which $f(x) = x^3 + x^2 - x - 1$ is concave up.

- a) $(-1, 1)$
- b) $(-1, \frac{1}{3})$
- c) $(-\infty, -1)$ and $(\frac{1}{3}, \infty)$
- d) $(-\frac{1}{3}, \infty)$
- e) $(-\infty, -\frac{1}{3})$

6 pts 10. Find the equations of all horizontal and vertical asymptotes of the function $f(x) = \frac{x^2 - x - 2}{x^2 + 4x + 3}$.

- a) $y = 0$ and $x = -1$
- b) $y = 1$, $x = -3$, and $x = -1$
- c) $y = 2$ and $x = -3$
- d) $y = 2$, $y = -1$, $x = -3$, and $x = -1$
- e) $y = 1$ and $x = -3$

6 pts 11. The product of two positive real numbers is 3. Find the smallest possible sum of twice the first number and three times the second number.

a) $6\sqrt{2}$

b) $5\sqrt{3}$

c) 11

d) 9

e) 7

6 pts 12. Find $\int \frac{3t - 2}{\sqrt{t}} dt$.

a) $2t^{3/2} + \frac{4}{3}t^{-3/2} + C$

b) $-6t^{-1/2} + \frac{4}{3}t^{-3/2} + C$

c) $\frac{9t^2 - 12t}{4t^{3/2}} + C$

d) $-6t^{-1/2} - 4t^{1/2} + C$

e) $2t^{3/2} - 4t^{1/2} + C$

- 6 pts 13. Let f be a continuous function on $(-\infty, \infty)$, such that $\int_{-1}^3 f(x) dx = -5$ and $\int_{-1}^1 f(x) dx = 2$.

Compute $\int_1^3 f(x) dx$.

- a) 7
- b) 3
- c) -3
- d) -7
- e) -10

- 6 pts 14. Find $\frac{dy}{dx}$ where $y = \int_1^{\tan x} \frac{1}{1+t^4} dt$.

- a) $\frac{1}{1 + \tan^4 x}$
- b) $\frac{\sec^2 x}{1 + \tan^4 x}$
- c) $\frac{1}{1 + \tan^4 x} - 1$
- d) $\frac{1}{1 + \tan^4 x} - \frac{1}{2}$
- e) $\frac{\sec^2 x}{1 + \tan^4 x} - \frac{1}{2}$

6 pts 15. Evaluate $\int_0^2 \frac{t}{\sqrt{t^2 + 4}} dt$.

a) $2\sqrt{2} - 2$

b) $8\sqrt{2} - 8$

c) $\sqrt{2}$

d) $4\sqrt{2}$

e) $8\sqrt{2}$

6 pts 16. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \sqrt{x-1}$, $x = 2$, $x = 5$, and $y = 0$ about the x -axis.

a) $\frac{15}{2}$

b) $\frac{15\pi}{2}$

c) $\frac{16\pi}{3}$

d) $\frac{14}{3}$

e) $\frac{14\pi}{3}$

6 pts 17. Let $f(x) = \sin^2 x$ on $[0, \frac{3\pi}{4}]$. Estimate $\int_0^{\frac{3\pi}{4}} f(x) dx$ using a Riemann sum with three equal length subintervals and where the sample points are right endpoints.

a) 0

b) $\frac{\pi}{8}$

c) $\frac{\pi}{4}$

d) $\frac{3\pi}{8}$

e) $\frac{\pi}{2}$

6 pts 18. Find $\int_0^3 |x - 1| dx$.

a) $\frac{3}{2}$

b) $\frac{5}{2}$

c) $\frac{9}{2}$

d) $\frac{11}{2}$

e) $\frac{15}{2}$

10 pts

19. Evaluate $\int_1^4 \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$.

16 pts 20. A region in the xy -plane is enclosed by the curves $x = 1 - y^2$ and $x = 2y^2 - 2$.

a) (3 points) Find the x and y -coordinates of the points of intersection of these curves.

b) (5 points) Draw the region. Label the curves and their points of intersection.

c) (8 points) Calculate the area of the region.

16 pts 21. A region is bounded by the curves $y = \sin(x) + 1$ and $y = 1$ where $0 \leq x \leq \pi$. Set up integrals which measure the volumes of the solids obtained by rotating this region about the indicated axes. DO NOT EVALUATE THE INTEGRALS.

a) Rotate around the x -axis.

b) Rotate around the y -axis.