

Name _____ ID # _____ Section # _____

Instructor _____

This examination consists of 23 questions. The first 19 are 6-point multiple choice questions, the next one is “short answer” and the last three are free response questions (for which partial credit is possible). The point value for each question appears to the left of the question number. There are 150 total points.

Please record your answers to the multiple choice questions by circling the corresponding letter. If your instructor is using scantron forms, also be sure to fill in the appropriate circle on your scantron form using a number 2 pencil. Present your work clearly for the free response/partial credit questions. **No credit will be given for unsupported answers in the free response/partial credit section.** The use of calculators, books, or notes is prohibited.

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| <p style="text-align: center;">Check the examination booklet before you start. There should be 23 questions on 15 pages.</p> |
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DO NOT WRITE IN THE BLANKS BELOW.

MC. _____(114)

20. _____(6)

21. _____(12)

22. _____(8)

23. _____(10)

TOTAL _____(150)

6 pts 1. Find all the horizontal and vertical asymptotes of the function

$$y = \frac{12 + 2x - 2x^2}{9 - x^2}$$

a) Horizontal: $y = \frac{4}{3}$; Vertical: $x = -3, x = 3$

b) Horizontal: $y = -2$; Vertical: $x = -3, x = 3$

c) Horizontal: $y = \frac{4}{3}$; Vertical: $x = -3$

d) Horizontal: $y = 2$; Vertical: $x = -3, x = 3$

e) Horizontal: $y = 2$; Vertical: $x = -3$

6 pts 2. Find

$$\lim_{x \rightarrow 0} \frac{2 \cos x - 2}{\sqrt{\cos x + 3} - 2}$$

a) 0

b) $\frac{1}{2}$

c) 6

d) 8

e) The limit does not exist.

6 pts 3. Let $f(x) = \frac{1}{x^3}$. Find $f''(-1)$.

- a) 12
- b) -12
- c) -1
- d) $-\frac{1}{6}$
- e) 1

6 pts 4. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose?

- a) 100,000 m²
- b) 80,000 m²
- c) 90,000 m²
- d) 60,000 m²
- e) 125,000 m²

6 pts 5. Solve the following initial value problem

$$\frac{dy}{dx} = \sin(\pi x), \quad y(1) = 3.$$

a) $y = -\frac{1}{\pi} \cos(\pi x) + 3 - \frac{1}{\pi}$

b) $y = -\frac{1}{\pi} \cos(\pi x) + 3$

c) $y = -\pi \cos(\pi x) + 3 - \pi$

d) $y = \pi \cos(\pi x) + 3$

e) $y = \pi \cos(\pi x)$

6 pts 6. Determine the open interval(s) on which $f(x) = x^3 - 6x + 11$ is decreasing.

a) $(-\sqrt{2}, \sqrt{2})$

b) $(-\sqrt{2}, 0)$

c) $(\sqrt{2}, \infty)$

d) $(-\infty, \sqrt{2})$

e) $(-\infty, -\sqrt{2})$ and $(\sqrt{2}, \infty)$

6 pts 7. Evaluate

$$\int_0^1 x^5 \sqrt{7x^6 + 9} \, dx.$$

- a) $\frac{64}{63}$
- b) $\frac{37}{63}$
- c) $\frac{27}{63}$
- d) $\frac{1}{63}$
- e) $\frac{42}{63}$

6 pts 8. Route Y is a highway which runs north and south, and route X is a highway which runs east and west. The two highways intersect. A Volkswagen Jetta is traveling southbound on route Y away from the intersection at a steady rate of 50 miles per hour. A Honda Accord is traveling westbound on route X toward the intersection at a steady rate of 60 miles per hour. At what rate (in miles per hour) is the distance between the two cars changing at the moment when Jetta is 3 miles south of the intersection and the Accord is 4 miles east of the intersection?

- a) -110
- b) -78
- c) -18
- d) 18
- e) 78

6 pts 9. Find the equation of the tangent line to the curve $y = x \cos x$ at $x = 0$.

a) $y = x + 1$

b) $y = x$

c) $y = \sin x$

d) $y = -x$

e) $y = (\cos x - x \sin x)x$

6 pts 10. Find the x -coordinate(s) of the critical point(s) of $f(x) = x^{5/3} - 20x^{2/3} + 2003$.

a) $x = 0$ only.

b) $x = 8$ only.

c) $x = 0$ and $x = 8$.

d) $x = -4$, $x = 0$, and $x = 8$.

e) $x = -4$, and $x = 0$.

6 pts 11. Find $\frac{dy}{dx}$ if $x^3 + y^3 + xy^2 = \pi^2$.

a) $\frac{dy}{dx} = -\frac{3x^2 + y^2}{3y^2 + 2xy}$

b) $\frac{dy}{dx} = \frac{2\pi - 3x^2 + y^2}{3y^2 + 2xy}$

c) $\frac{dy}{dx} = 3x^2 + 3y^2 + 2y - 2\pi$

d) $\frac{dy}{dx} = 3x^2 + 3y^2 + 2y$

e) $\frac{dy}{dx} = -\frac{3x^2}{3y^2 + 2y}$

6 pts 12. Find $\lim_{x \rightarrow 0^-} \frac{x}{|x|}$.

a) -1

b) 0

c) 1

d) ∞

e) $-\infty$

- 6 pts 13. Suppose that $f(x)$ is continuous and $\int_{-1}^0 f(x) dx = 4$ and $\int_{-1}^2 f(x) dx = 5$. Determine
- $$\int_2^0 f(x) dx.$$

- a) 9
- b) -9
- c) 1
- d) -1
- e) 0

- 6 pts 14. Find the derivative $\frac{dy}{dx}$ of the function

$$y = \int_{\pi}^{\sqrt{x}} \frac{\cos t}{t} dt.$$

- a) $\frac{\cos \sqrt{x}}{2x}$
- b) $\frac{-\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}}{x}$
- c) $\frac{-\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}}{2x\sqrt{x}}$
- d) $\frac{\cos \sqrt{x}}{\sqrt{x}}$
- e) $\frac{\cos \sqrt{x}}{x} + \frac{1}{\pi}$

6 pts 15. Use the substitution $u = \sin 2x$ to simplify the following integral:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\cos 2x}{\sin^3 2x} dx$$

a) $-\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} u^{-3} du$

b) $2 \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} u^{-3} du$

c) $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} u^{-3} du$

d) $2 \int_{\frac{\sqrt{3}}{2}}^1 u^{-3} du$

e) $\frac{1}{2} \int_{\frac{\sqrt{3}}{2}}^1 u^{-3} du$

6 pts 16. Evaluate the definite integral $\int_0^{\frac{\pi}{4}} \sec \theta \tan \theta d\theta$.

a) $\frac{\sqrt{2}}{2}$

b) 1

c) $\frac{\sqrt{2}}{2} - 1$

d) $\sqrt{2} - 1$

e) $\sqrt{2}$

6 pts 17. Find the total area of the region between the curve $f(x) = x^2 - 4$ and the x -axis for $0 \leq x \leq 4$.

a) $\frac{16}{3}$

b) $\frac{32}{3}$

c) 11

d) 16

e) 32

6 pts 18. Evaluate $\int \cos^2 x \, dx$.

a) $\sin^2 x + C$

b) $1 - \sin^2 x + C$

c) $\frac{x}{2} - \frac{\sin 2x}{4} + C$

d) $\frac{x}{2} + \frac{\sin 2x}{4} + C$

e) $\frac{1}{3} \cos^3 x + C$

6 pts 19. Find $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$.

- a) 0
- b) 1
- c) $\cos x$
- d) $\sin x$
- e) The limit does not exist.

6 pts 20. Consider the function $f(x) = (x - 10)^2$ over the interval $I = [2, 10]$.

a) Partition I into four equal subintervals. Compute the corresponding Riemann sum approximation of $\int_2^{10} f(x) dx$ using the righthand endpoint of each subinterval.

b) Is your answer in part (a) a lower estimate or upper estimate? Circle the correct choice:

(i) It is a lower estimate.

(ii) It is an upper estimate.

(iii) It is impossible to tell whether it is an upper or lower estimate from the given information.

12 pts 21. Let R be the interior region of the first quadrant bounded by the curve $y = x^3$ and the line $y = 4x$.

a) Write down an integral to express the volume of the solid generated by rotating R about the x -axis. DO NOT EVALUATE THE INTEGRAL.

b) Write down an integral to express the volume of the solid generated by rotating R about the y -axis. DO NOT EVALUATE THE INTEGRAL.

8 pts 22. Evaluate

$$\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx.$$

10 pts 23. Let R be the region of the plane enclosed by the curve $x + y^2 = 4$ and the line $x - y = 2$.

a) Sketch the the region on the axes provided below. Be sure to label all intersection points.

b) Write down an expression involving integral(s) which represents the area of R . DO NOT EVALUATE THE INTEGRAL(S).