

Name _____ ID # _____ Section # _____

There are 16 multiple-choice questions, 8 True/False questions, and 3 partial credit questions. For the partial credit problems you **must present your work clearly and understandably; no credit will be given for unsupported answers.** For True/False and multiple-choice problems, please circle the correct answer in each question.

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

THERE ARE 20 PROBLEMS ON 13 PAGES, INCLUDING THIS ONE.
CHECK YOUR BOOKLET NOW.

The area below is for the instructor's use.

MC (96)

T/F (16)

18 (10)

19 (14)

20 (14)

Total (150)

1. (6 pts.) $\lim_{x \rightarrow 5} \frac{x^2}{(x-5)^3} =$

- a) 5
- b) 25
- c) $-\infty$
- d) ∞
- e) The limit does not exist.

2. (6 pts.) $\lim_{x \rightarrow 2^-} \frac{|x-2|}{x^2-4} =$

- a) $\frac{-1}{4}$
- b) 0
- c) 1
- d) $\frac{1}{4}$
- e) The limit does not exist.

3. (6 pts.) $\lim_{x \rightarrow 3} \frac{x^2 - 3x}{\sqrt{x+1} - 2} =$

- a) 12
- b) 9
- c) 3
- d) 1
- e) The limit does not exist.

4. (6 pts.) What is the instantaneous rate of change of the function $f(x) = \tan(\pi x^2)$ at $x = \frac{1}{2}$?

- a) 0
- b) 1
- c) $\sqrt{2}\pi$
- d) 2π
- e) $-\pi$

5. (6 pts.) What are the horizontal and the vertical asymptotes of the function

$$f(x) = \frac{2x(x+2)^2(x-1)}{x^2(x-1)}?$$

- a) H.A. none; V.A. none
- b) H.A. $y = 2$; V.A. none
- c) H.A. $y = 2$; V.A. $x = 0$ and $x = 1$
- d) H.A. $y = 2$; V.A. $x = 0$
- e) H.A. none; V.A. $x = 0$

6. (6 pts.) Let $F(x) = f(g(x))$. Suppose $f(2) = 5$, $f'(2) = -1$, $f(3) = 7$, $f'(3) = 2$, $g(2) = 3$, $g'(2) = 4$, $g(5) = 6$, and $g'(5) = -2$. Find $F'(2)$.

- a) -4
- b) 8
- c) 2
- d) 7
- e) -10

7. (6 pts.) A right triangle has base b and height h . Suppose the base is decreasing at a rate of $\frac{1}{2} m/sec$, while the height is increasing at a rate of $2 m/sec$. At what rate is the triangle's area increasing when $b = 4$ and $h = 5$?

a) $-\frac{1}{2} m^2/sec$

b) $\frac{11}{4} m^2/sec$

c) $\frac{21}{4} m^2/sec$

d) $\frac{3}{2} m^2/sec$

e) $0 m^2/sec$

8. (6 pts.) Suppose $f(x)$ is a differentiable function and $f'(x) = x^3(x+2)(x-2)^2$, then $f(x)$ has a local minimum at

a) $x = -2$ only

b) $x = 0$ only

c) $x = 0, x = 2$

d) $x = -2, x = 2$

e) $x = -2, x = 0$, and $x = 2$

9. (6 pts.) Suppose x and y are two positive numbers such that $xy = 6$, what is the minimum value of the sum $2x + 3y$?

- a) 2
- b) 3
- c) 5
- d) 12
- e) 15

10. (6 pts.) Suppose $f(x)$ is a continuous function and that

$$\int_0^3 f(x) dx = 2, \quad \int_6^9 f(x) dx = -1, \quad \int_0^9 f(x) dx = 5$$

then what is $\int_6^3 f(x) dx$?

- a) -4
- b) -2
- c) 4
- d) 6
- e) The value cannot be determined by the information given.

11. (6 pts.) $\int_{-2}^2 (|x| + 1) dx =$

- a) 0
- b) 2
- c) 4
- d) 8
- e) The definite integral does not exist because the integrand is not differentiable at $x = 0$.

12. (6 pts.) $\int_0^{\frac{\pi}{4}} \tan^2 x dx =$

- a) 0
- b) 1
- c) $1 - \frac{\pi}{4}$
- d) $\sqrt{2}$
- e) $\frac{1}{3}$

13. (6 pts.) A particle is moving along a straight line with a velocity given by $v(t) = t^2 - 2t + 4$. What is the net change of its position between $t = 1$ and $t = 3$?

- a) $-\frac{14}{3}$
- b) -2
- c) 4
- d) 12
- e) $\frac{26}{3}$

14. (6 pts.) Evaluate $\frac{d}{dx} \int_{\pi}^{\sqrt{x}} t \cos^3(2t^2) dt$.

- a) $\sqrt{x} \cos^3(2x)$
- b) $\sqrt{x} \cos^3(4x)$
- c) $\frac{1}{4} \sqrt{x} \cos^4(2x)$
- d) $\frac{1}{2} \cos^3(2x)$
- e) $\frac{1}{8} \cos^4(4x)$

15. (6 pts.) When using the substitution $u = \sqrt{x}$, the definite integral $\int_1^{25} \frac{\sin^3 \sqrt{x}}{\sqrt{x}} dx$ becomes

a) $\frac{1}{2} \int_1^{25} \sin^3 u \, du$

b) $2 \int_1^5 \sin^3 u \, du$

c) $\int_1^{25} \sin^3 u \, du$

d) $\int_1^5 u \sin^3 u \, du$

e) $\int_1^5 \frac{\sin^3 u}{u} \, du$

16. (6 pts.) The average (mean) value of the function $f(x) = 3x^2 - 2x + 2$ on the interval $[0, 3]$ is

a) 7

b) 8

c) $\frac{23}{4}$

d) 21

e) 24

17. (16 pts., 2 pts. each) Suppose f is a function differentiable everywhere on $(-\infty, \infty)$, such that $f(0) = 3$, $f'(0) = 0$, $f(2) = -1$, $f'(2) = 5$, $f(10) = 10$, and $f'(10) = -2$. True or False:

- a) T F f must be continuous everywhere on $(-\infty, \infty)$.
- b) T F f must have at least 2 real roots.
- c) T F The average rate of change of f , between $x = 0$ and $x = 10$, is $-\frac{1}{5}$.
- d) T F There must exist a point $x = c$, $0 < c < 2$, such that $f'(c) = -2$.
- e) T F $\lim_{x \rightarrow 0^-} f(x) = 0$.
- f) T F The graph $y = f(x)$ might not have a tangent line at $x = 0$.
- g) T F The point $x = 0$ is a critical point of f .
- h) T F The graph $y = f(x)$ might have a vertical asymptote somewhere.

18. (10 pts.) Use the substitution method of integration to evaluate the definite integral

$$\int_0^4 \frac{(2 + \sqrt{x})^3}{4\sqrt{x}} dx$$

19. (14 pts.) Consider the region(s) in the xy -plane enclosed by the curves $y = x^3$ and $y = 3x$.
- a) (3 points) Find the x and y -coordinates of the points of intersection of these curves.
- b) (3 points) Draw the region(s). Label the curves and their points of intersection.
- c) (8 points) Calculate the total area of the region(s).

20. (14 pts.) Consider the first quadrant region bounded by the curves $y = x^3$ and $y = 3x$. Set up integrals which measure the volumes of the solids obtained by rotating this region about the indicated axes. **DO NOT EVALUATE THE INTEGRALS.**

a) Rotate about the x -axis.

b) Rotate about the y -axis.