

Name _____ ID # _____ Section # _____

There are 10 multiple choice questions, 5 True/False questions, and 3 free response questions. **To receive full credit for free response questions (problems 12, 13, and 14) all work must be shown.**

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

THERE ARE 14 PROBLEMS ON 10 PAGES, INCLUDING THIS ONE. CHECK YOUR BOOKLET NOW.

The space below is for the instructor's use.

MC _____

T/F _____

12. _____

13. _____

14. _____

Total _____

1. (5 pts.) If $f(x) = \sin^2(x^3 + \pi^3)$, then $f'(x)$ is

- a) $2 \sin(x^3 + \pi^3) \cos(x^3 + \pi^3)$
- b) $\cos^2(x^3 + \pi^3)$
- c) $2(3x^2 + 3\pi^2) \sin(x^3 + \pi^3)$
- d) $6x^2 \sin(x^3 + \pi^3) \cos(x^3 + \pi^3)$
- e) $2 \sin(x^3 + \pi^3) \cos(x^3 + \pi^3)(3x^2 + 3\pi^2)$

2. (5 pts.) If $f(\theta) = 2 \tan \theta$, then $f''(\pi/4)$ is

- a) 8
- b) 16
- c) 4
- d) 1
- e) 6

3. (5 pts.) Find the value of c in the conclusion of the Mean Value Theorem for $f(x) = \sqrt{x-2}$ on $[2, 6]$.

- a) 0
- b) 3
- c) 5
- d) $1/2$
- e) $5/4$

4. (5 pts.) Suppose $f(3) = 2$, and $f'(3) = 1/4$. Estimate $f(3.4)$ using the linear approximation of f at 3.

- a) 2.1
- b) 4.1
- c) 1.4
- d) 2
- e) 2.4

5. (5 pts.) Determine all vertical and horizontal asymptotes of the function $f(x) = \frac{x^2 - 9}{2x^2 + 6x}$.

- a) Horizontal: $y = 1$ Vertical: $x = -3$
- b) Horizontal: $y = 1$ Vertical: $x = -6$
- c) Horizontal: $y = 1/2$ Vertical: $x = 0$ and $x = -6$
- d) Horizontal: $y = 1/2$ Vertical: $x = 0$
- e) Horizontal: $y = 1/2$ Vertical: $x = 0$ and $x = -3$

6. (5 pts.) Find the absolute minimum m and the absolute maximum M of the function $f(x) = 3x^{2/3} - 2x$ on $[-1,1]$.

- a) $m = 0, \quad M = 5$
- b) $m = 0, \quad M = 1$
- c) $m = 1, \quad M = 5$
- d) $m = 0$. There is no absolute maximum value.
- e) There is no absolute minimum value. $M = 5$.

7. (5 pts.) Find $\frac{dy}{dx}$, if x and y are related by $2x^3 + x^2y + y^3 = 8000$.

a) $\frac{6x^2 + 2xy}{x^2 + 3y^2}$

b) $-\frac{6x^2}{x^2 + 3y^2}$

c) 1

d) $-\frac{6x^2 + 2xy}{x^2 + 3y^2}$

e) $-\frac{6x^2}{2x + 3y^2}$

8. (5 pts.) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 2} - x)$ is

a) 0.

b) 1.

c) 2.

d) ∞ .

e) not defined.

9. (5 pts.) Suppose $f'(x) = \frac{(x-2)^2 x^5}{x+1}$. Find the intervals on which f is increasing.

- a) $(-\infty, -1)$
- b) $(-\infty, -1) \cup (0, 2) \cup (2, \infty)$
- c) $(-1, 0) \cup (2, \infty)$
- d) $(-\infty, -1) \cup (2, \infty)$
- e) $(-1, 0) \cup (0, -2)$

10. (5 pts.) Given the function $F(x) = f(g(x))$, find $F'(2)$ if $f(2) = 3$, $f(3) = 4$, $f'(5) = 8$, $f'(2) = 6$, $g(2) = 5$, $g(6) = 1/3$, $g'(5) = 4$ and $g'(2) = 2$.

- a) 16
- b) 12
- c) 6
- d) 24
- e) 3

11. (10 pts. 2 pts. each) True or False: **(Circle the appropriate letter.)**

- a) T F If $f''(c) = 0$ then the graph of f must have an inflection point at $x = c$.
- b) T F If $f'(c) = 0$ then f has a local extremum at $x = c$.
- c) T F If $f'(c) = 0$ and f'' is continuous and positive at c , then f has a local minimum at $x = c$.
- d) T F If $f'(x) = g'(x)$ on an interval, then $f(x) = g(x) + C$ on the interval, where C is a constant.
- e) T F If f is a continuous on a closed interval, then it has an absolute maximum on that interval.

12. (16 pts.) Consider the function $f(x) = 2x^{5/3} - 5x^{4/3}$.

(a) Identify the intervals on which $f(x)$ is increasing and on which $f(x)$ is decreasing.

(b) For what value(s) of x does $f(x)$ have a local maximum and for what value(s) does it have a local minimum?

(c) Identify the intervals on which the graph of $f(x)$ is concave up and those on which graph of $f(x)$ is concave down.

(d) What are the inflection points of $f(x)$, if any.

13. (12 pts.) A 6-foot tall man walks at the rate of 4 ft/sec away from the base of a street light 12 feet above the ground. At what rate is the length of his shadow changing when he is 20 feet away from the base of the light?

14. (12 pts.) An open rectangular box of maximum volume is to be made using an 8 inches by 15 inches piece of cardboard by cutting squares from the corners and folding up sides. What are the dimensions of the box of largest volume you can make this way?