MATH 140A MIDTERM EXAMINATION II April 6, 2004

Name_____ ID #_____ Section #_____

There are 8 multiple-choice questions, 8 True/False questions, and 4 partial credit questions. For the partial credit problems you **must present your work clearly and understandably; no credit will be given for unsupported answers. For True/False and multiple-choice problems, please circle the correct answer in each question.**

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

THERE ARE 13 PROBLEMS ON 10 PAGES, INCLUDING THIS ONE.
CHECK YOUR BOOKLET NOW.

The area below is for the instructor's use.

MC ..... (40)

T/F ..... (16)

10 ..... (8)

11 ..... (14)

12 ..... (12)

13 ..... (10)

Total ..... (100)

1. (5 pts.) The slope of the line tangent to the graph given by $y^3 - x^3 = 9$, at the point $(-1, 2)$ is

   a) $\frac{1}{4}$

   b) 0

   c) $-\frac{1}{2}$

   d) -4

   e) The tangent line does not exist at this point.

2. (5 pts.) Suppose $x = \cot y$, find $\frac{dy}{dx}$.

   a) $x^2 - 1$

   b) $\frac{1}{\sqrt{1+x^2}}$

   c) $\frac{1}{\sqrt{1-x^2}}$

   d) $\frac{-1}{1+x^2}$

   e) $\frac{-1}{x\sqrt{x^2-1}}$

3. (5 pts.) The graph of $y = x^2 + \dfrac{1}{x^2}$ has

a) Local maximums at $x = -1$ and $x = 1$, no local minimums.

b) Local maximum at $x = -1$, local minimum at $x = 1$.

c) No local maximums, local minimums at $x = -1$ and $x = 1$.

d) Local maximum at $x = 0$, local minimums at $x = -1$ and $x = 1$.

e) Local maximums at $x = -1$ and $x = 1$, local minimum at $x = 0$.

4. (5 pts.) On the interval $[0, \pi]$, what is the critical point(s) of $f(x) = \sin(x)\cos(x)$?

a) $x = 0$

b) $x = \dfrac{\pi}{2}$ and $x = \pi$

c) $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{4}$

d) $x = \dfrac{\pi}{4}$ and $x = \dfrac{\pi}{2}$

e) $x = 0$, $x = \dfrac{\pi}{2}$, and $x = \pi$

5. (5 pts.) Find the value of $x = c$ that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \dfrac{1}{x^3}$ on the interval $[1, 2]$.

   a) $\sqrt{\dfrac{24}{7}}$

   b) $\sqrt[4]{\dfrac{24}{7}}$

   c) $-\dfrac{7}{8}$

   d) $-\sqrt[3]{\dfrac{8}{7}}$

   e) The Mean Value Theorem does not apply since $f(x)$ is not continuous at $x = 0$.

6. (5 pts.) If the graph of $f(x) = x^3 + cx^2 - 6x + 2$ has a point of inflection at $x = 0$, then $c =$

   a) -3

   b) 0

   c) 3

   d) 6

   e) The graph does not have a point of inflection.

7. (5 pts.) What are the horizontal and the vertical asymptotes of the function $f(x) = \dfrac{(2x+1)(2-x)}{(x-2)(3x-5)}$?

   a) H.A. $y = \dfrac{-2}{3}$; V.A. $x = \dfrac{5}{3}$

   b) H.A. $y = \dfrac{2}{3}$; V.A. $x = \dfrac{5}{3}$ and $x = 2$

   c) H.A. $y = \dfrac{-4}{3}$; V.A. $x = \dfrac{5}{3}$ and $x = 2$

   d) H.A. $y = \dfrac{2}{3}$; V.A. $x = \dfrac{5}{3}$

   e) H.A. none; V.A. $x = \dfrac{5}{3}$ and $x = 2$

8. (5 pts.) Suppose $f(5) = 2$ and $f'(5) = -2$. Then the linear approximation $L(4.8)$ of $f(x)$ at $x = 4.8$ is

   a) 1.2

   b) 1.6

   c) 2

   d) 2.4

   e) 2.8

9. (16 pts., 2 pts. each) True or False:

a) T F A differentiable function might not have any critical point.

b) T F If $f'(c)$ does not exist, then $x = c$ cannot be a critical point.

c) T F $\lim_{x\to\infty} \dfrac{1+2x-x^3}{2x^2-x+4} = -\infty$.

d) T F If $f'(c) = 0$, then the graph of $y = f(x)$ must have a horizontal tangent line at $(c, f(c))$.

e) T F If $f''(c) = 0$, then $x = c$ is a point of inflection of $y = f(x)$

f) T F If $f'(c) = 0$ and $f''(c) > 0$, then $f(x)$ must have a local minimum at $x = c$.

g) T F If a function $f(x)$ is negative and increasing on an interval I, then $g(x) = f^3(x)$ is also negative and increasing on I.

h) T F If $\int f(x)dx = F(x)$ and $\int g(x)dx = G(x)$, then $\int [f(x)g(x)]dx = F(x)G(x)$.

10. (8 pts.)

a) Differentiate the function below. Please do not simplify your answers.
$f(x) = \sec^{100}(\cos(x^5 - 3x^3 + 1))$

b) Integrate $\displaystyle\int (-2x^4 + \sqrt[6]{x} - \csc^2(x) + \sin(x) + 5)\ dx$

11. (14 pts.) Lizzie McGuire is flying a kite at a steady height of $300ft$. The wind is carrying the kite horizontally away from her at a rate of $10ft/sec$.

a) How fast should Lizzie let out the string when the kite is $500ft$ away (that is, after $500ft$ of string has been let out)?

b) How fast is the angle θ, between the string and the ground, changing at the same moment?

12. (12 pts.) For the function $f(x) = x^4 - 8x^2 + 12$

a) Find all critical points of $f(x)$.

b) Determine if each critical point is a local maximum or a local minimum.

c) Find the absolute maximum and the absolute minimum points of $f(x)$ on the interval $[-3, 1]$.

13. (10 pts.) Lizzie and Gordo are building a $27ft^3$ aquarium for their school project. The tank is to have a square base and no top. The base is to be made with stainless steel costing \$6 per ft^2, and the sides are to be made with plexiglass costing \$3 per ft^2. Find the dimensons of the aquarium that would minimize the cost. Verify that your answer indeed gives the minimum cost.