

Name _____ ID # _____ Section # _____

There are 12 multiple-choice questions, 9 True/False questions, and 2 partial credit / short answer questions. For the partial credit problems you **must** present your work clearly and understandably; no credit will be given for unsupported answers. For each multiple-choice problem **five** possible answers are given, only one of which is correct. **Circle** the correct answer in your exam booklet **and blacken** the corresponding space on the **scantron form**. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle). For True/False problems, please circle the correct answer in each question.

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

THERE ARE 15 PROBLEMS ON 10 PAGES, INCLUDING THIS ONE.
CHECK YOUR BOOKLET NOW.

The area below is for the instructor's use.

MC (60)

T/F (18)

14 (10)

15 (12)

Total (100)

1. (5 pts.) A polynomial function always has all the properties listed below, EXCEPT

- a) Its domain is $(-\infty, \infty)$.
- b) It is continuous everywhere.
- c) Its graph has exactly one y -intercept.
- d) Its graph does not have any vertical asymptote.
- e) Its range is $(-\infty, \infty)$.

2. (5 pts.) Suppose $f(x) = x^3 + 1$ and $h \neq 0$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$.

- a) $3x^2 + 3hx + h^2$
- b) $3x^2$
- c) $\frac{x^3 + 3hx^2 + 3h^2x + h^3}{h}$
- d) h^2
- e) $\frac{1}{h}$

3. (5 pts.) $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x^2 - 5x + 6} =$

- a) 7
- b) 0
- c) 1
- d) $+\infty$
- e) $-\infty$

4. (5 pts.) $\lim_{x \rightarrow 1^-} \frac{x + 3}{x^2(x - 1)} =$

- a) 0
- b) 3
- c) 4
- d) $-\infty$
- e) $+\infty$

5. (5 pts.) $\lim_{x \rightarrow 4} \frac{\sqrt{x^2 + 9} - 5}{x - 4} =$

- a) 0
- b) $\frac{-1}{5}$
- c) $\frac{4}{5}$
- d) $+\infty$
- e) $-\infty$

6. (5 pts.) $\lim_{x \rightarrow -2} \frac{x + 5}{(x + 2)^2} =$

- a) $-\infty$
- b) -2
- c) 3
- d) 0
- e) $+\infty$

7. (5 pts.) $\lim_{x \rightarrow 3^+} \frac{9 - x^2}{|3 - x|} =$

- a) -6
- b) 0
- c) 6
- d) $-\infty$
- e) $+\infty$

8. (5 pts.) If

$$f(x) = \begin{cases} -x^2 + c & , \quad x < -1 \\ cx + 3 & , \quad x \geq -1 \end{cases}$$

is a continuous function defined on $(-\infty, \infty)$, then what is the value of c ?

- a) -1
- b) 2
- c) 3
- d) 4
- e) Not enough information is given to determine the value of c .

9. (5 pts.) According to the Intermediate Value Theorem, which of the polynomials below must have a real root on the interval $(-1, 0)$?

a) $x^3 - 3x^2 + 3x - 4$

b) $x^3 - 3x^2 + 3x - 1$

c) $x^3 + 2x^2 + 4x - 1$

d) $x^3 + 2x^2 + 4x + 2$

e) $x^3 + 2x^2 + 4x + 5$

10. (5 pts.) Which of the equations below represents the line tangent to the circle given by $x^2 + y^2 = 5$, at the point $(1, -2)$?

a) $y = -\frac{1}{2}x - \frac{3}{2}$

b) $y = -2x - 4$

c) $y = -2x$

d) $y = \frac{1}{2}x + \frac{3}{2}$

e) $y = \frac{1}{2}x - \frac{5}{2}$

11. (5 pts.) Which of the following is an infinite discontinuity (where it has a vertical asymptote) on the graph of the function

$$f(x) = \frac{x(x+1)(x+3)^2}{x^2(x^2-1)(x+3)}?$$

I. $x = -3$, II. $x = -1$, III. $x = 0$, IV. $x = 1$.

- a) IV only
b) I and III only
c) III and IV only
d) I, II, and III only
e) I, II, III, and IV
12. (5 pts.) The average rate of change of the function $f(x) = x^3 - 5x + 4$, on the interval $[-2, 2]$, is

- a) -4
b) -1
c) 0
d) 1
e) 4

13. (18 pts., 2 pts. each) True or False:

a) T F For all real numbers a and b , $a^2 - b^2 = (a + b)(a - b)$.

b) T F For all real numbers a and b , $|a + b| = |a| + |b|$.

c) T F For all $a, b \geq 0$, $\sqrt{a^2 + b^2} = a + b$.

d) T F For all $a > 0$, $a^{x+y} = a^x a^y$.

e) T F If $f(x)$ is not continuous at $x = c$, then $f(c)$ does not exist.

f) T F If $\lim_{x \rightarrow 7} f(x) = L$ and $\lim_{x \rightarrow 7} g(x) = M$, then $\lim_{x \rightarrow 7} (3f(x) - 2g(x)) = 3L - 2M$.

g) T F If $\lim_{x \rightarrow c^-} f(x) \neq \lim_{x \rightarrow c^+} f(x)$, then $f(x)$ cannot be continuous at $x = c$.

h) T F The graph of a rational function might have one or more discontinuities.

i) T F If $f(x)$ is a continuous function such that $f(1) = 7$ and $f(5) = -3$. Then there is always at least one point, $x = c$, on the interval $(1, 5)$ such that $f(c) = 6$.

14. (10 pts.) Suppose

$$f(x) = \begin{cases} x^2 + 4, & x \leq 0 \\ \frac{2x - 2}{(x - 1)(x - 3)}, & 0 < x \leq 4 \\ -\frac{4}{x - 6}, & x > 4 \end{cases}$$

For each of the points given below, determine whether $f(x)$ is continuous there. For each discontinuity found, determine its type.

At $x = 0$

Answer: _____

At $x = 1$

Answer: _____

At $x = 3$

Answer: _____

At $x = 4$

Answer: _____

At $x = 6$

Answer: _____

15. (12 pts.) Consider

$$f(x) = \frac{x^2 + ax + b}{cx + 8}$$

such that:

- I. the domain of f consists of all real numbers except $x = -4$ (it has a removable discontinuity there),
- II. the graph of f has an x -intercept of 2, and
- III. the graph of f has a y -intercept of -1 .

Find the values of a, b and c .

Answer: $a =$ _____ , $b =$ _____ , $c =$ _____ .