MATH 140A
EXAMINATION I
SEPTEMBER 23, 2002

NAME _____
STUDENT NUMBER _____
INSTRUCTOR _____
SECTION NUMBER _____

The examination consists of **15** problems: **11** multiple choice questions followed by **4** partial credit problems. For the partial credit problems you **must present your work clearly and understandably; no credit will be given for unsupported answers. For this exam calculators are not allowed and are not needed. For multiple choice problems, please circle the correct answer in each question.**

The point value for each question is shown next to each question in the left margin. At the end of the examination, the booklet will be collected.

**THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.**

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE **15** PROBLEMS ON **9** PAGES (INCLUDING THIS ONE).

M.C. _____

12. _____

13. _____

14. _____

15. _____

TOTAL _____

5 pts 1. Find the center and radius of the circle given by the equation

$$2x^2 + 2y^2 + 8x - 12y + 8 = 0$$

a) Center: $(-2, 3)$ ; Radius: 3
b) Center: $(2, 3)$ ; Radius: 3
c) Center: $(2, -3)$ ; Radius: 9
d) Center: $(4, -6)$ ; Radius: 3
e) Center: $(-4, 6)$ ; Radius: 9

5 pts 2. The average rate of change of $f(x) = 3x^2 + x + 2$ on the interval $(1, 2)$ is

a) $-\dfrac{6}{3}$
b) $-\dfrac{16}{3}$
c) 6
d) 10
e) 16

5 pts 3. For $f(x) = x^4$, the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ is

a) $\dfrac{x^4}{h}$
b) $\dfrac{x^4 + h}{h}$
c) $\dfrac{x^4 + 4hx^3 + 6h^2x^2 + 4h^3x + h^4}{h}$
d) $4x^3 + 6hx^2 + 4h^2x + h^3$
e) 1

5 pts 4. Evaluate $\lim_{x\to 2} \dfrac{x^2}{(x+1)^2(x-2)}$

a) $\dfrac{4}{9}$

b) 1

c) $\infty$

d) $-\infty$

e) Does not exist.

5 pts 5. Evaluate $\lim_{x\to 3} \dfrac{x+3}{(x-3)^2}$

a) 0

b) $\dfrac{1}{3}$

c) 1

d) $\infty$

e) Does not exist.

5 pts 6. Evaluate $\lim_{x\to 0} \dfrac{\sqrt{x^2+25}-5}{x^2}$

a) $\dfrac{1}{10}$

b) $\dfrac{1}{5}$

c) 1

d) 15

e) Does not exist.

5 pts 7. Evaluate $\lim_{x \to 1} \dfrac{x^2 + 2x - 3}{x^2 - 3x + 2}$

a) $-4$
b) $0$
c) $1$
d) $\infty$
e) Does not exist.

5 pts 8. Suppose $f(x)$ is a continuous function, and $(1, 3)$, $(2, 0)$, $(3, 5)$ are points on the graph of $f(x)$. What is the value of $\lim_{x \to 3^-} f(x)$?

a) $-3$
b) $0$
c) $1$
d) $5$
e) Not enough information is known to determine the limit.

5 pts 9. The Intermediate Value Theorem guarantees that $x^3 + 3x^2 - 12 = 0$ has a solution on which of the following intervals?

a) $(-2, -1)$
b) $(-1, 0)$
c) $(0, 1)$
d) $(1, 2)$
e) $(2, 3)$

5 pts 10. If the function

$$f(x) = \begin{cases} 3x^2 - 2c & , \quad x < 2 \\ \dfrac{cx - 2}{x + 2} & , \quad x \geq 2 \end{cases}$$

is continuous at $x = 2$, then what is the value of $c$?

a. 5

b. 6

c. 8

d. $\infty$

e. No such value can be found for $f(x)$ to be continuous at $x = 2$.

5 pts 11. Let $f(x) = 2x^2 - x$. Given that $f'(1) = 3$, what is the equation of the line tangent to the graph of $f(x)$ at $x = 1$?

a. $y = 3x$

b. $y = 3x - 1$

c. $y = 3x - 2$

d. $y = 3x - 3$

e. $y = 3x - 4$

14 pts 12. (2 pts each)

T F a) If both left-hand and right-hand limits of $f(x)$ exist at $x = c$, then $\lim_{x\to c} f(x)$ must exist.

T F b) If $f(x)$ is undefined at $x = c$, then $\lim_{x\to c} f(x)$ does not exist.

T F c) $\lim_{x\to c} f(x)$ may exist if $f(x)$ is discontinuous at $x = c$.

T F d) If $\lim_{x\to c} f(x)$ exists and $f(x)$ is defined at $x = c$, then $f(x)$ must be continuous at $x = c$.

T F e) If $f(x)$ is continuous at $x = c$, then $\lim_{x\to c^+} f(x) = f(c)$.

T F f) If $f(x)$ is differentiable at $x = c$, then $f(x)$ is also continuous at $x = c$.

T F g) If $f(x)$ is differentiable at $x = c$, then $\lim_{x\to c} f(x)$ exists.

8 pts 13. (2 pts each) Suppose $f, g$, and $h$ are 3 continuous functions, $f(x) \leq g(x) \leq h(x)$ for all $x$. If $f(1) = 2$, $f(4) = 5$, $f(6) = -2$, $h(1) = 4$, $h(4) = 5$, $h(6) = 6$. Answer the following questions:

a) (2 pts) $\lim_{x \to 1}[f(x) + h(x)] =$

b) (2 pts) $\lim_{x \to 4} g(x) =$

c) (2 pts) What is the name of the theorem you would use to answer part b) above?

d) (2 pts) True or False: There is some point $x = c$ on the interval $(1, 4)$ such that $f(c) = 3$.

T F

12 pts 14. For the function

$$f(x) = \begin{cases} \dfrac{x+2}{x^2-4}, & x < 3 \\ \dfrac{1}{x^2-4x}, & x > 3 \end{cases}$$

Find all points at which $f$ is discontinuous. For each discontinuity, determine the type of discontinuity and justify your answer. (To receive full credit you must show all work verifying the type of discontinuity.)

11 pts 15.

a) (3 pts) State the limit definition of the derivative.

$$f'(x) =$$

b) (8 pts) Use the limit definition of the derivative to compute the derivative of

$$f(x) = \sqrt{ax}\,, \text{ where } a \text{ is a positive constant.}$$

No credit will be given for using any other method to find your answer