Math 511, Autumn 2010

Problem Set 7, 2D Nonlinear systems -- numerical phase-planes

First, some notation. Let us define two binary operations $x \veebar y := \min(x, y)$ and $x \barwedge y := \max(x, y)$.

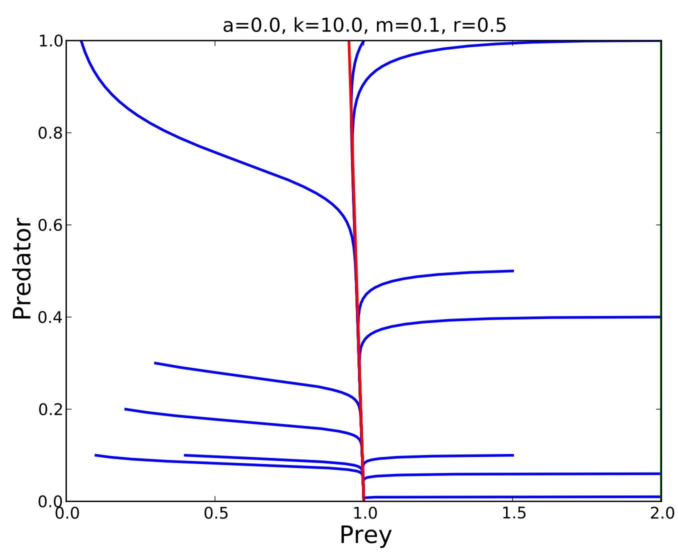
1. Consider the following system of ODE's that describes a predator-prey system:

$$\dot{x} = x(1 - x) - my[(rx) \veebar (k - kax + k^2 a/r)] \barwedge 0,$$
$$\dot{y} = my[(rx) \veebar (k - kax + k^2 a/r)] \barwedge 0 - my,$$

with $x \geq 0, y \geq 0$. Draw phaseplanes for this system for each of the following parameter sets.

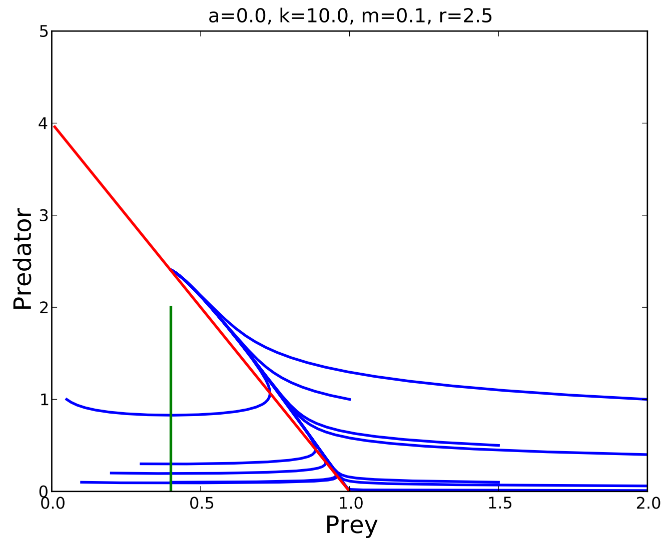
Answer:
In the following plots, the nullclines are shown in red and green, while the orbits are shown in blue. The dynamics of this system are quite complicated, with Hopf bifurcations, and global homoclinic-orbit bifurcations.

(a) $a = 0., k = 10., m = 0.1, r = 0.5$
Answer:



There is one globally-attracting stationary solution that is a stable node. There are no limit cycles.

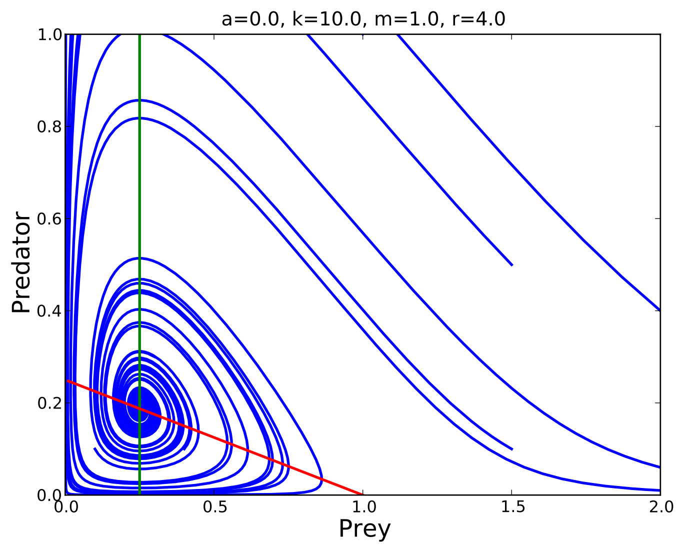
(b) $a = 0., k = 10., m = 0.1, r = 2.5$
Answer:



There is one saddle-point on the boundary and one stable focus on the interior.

(c) $a = 0., k = 10., m = 1, r = 4$

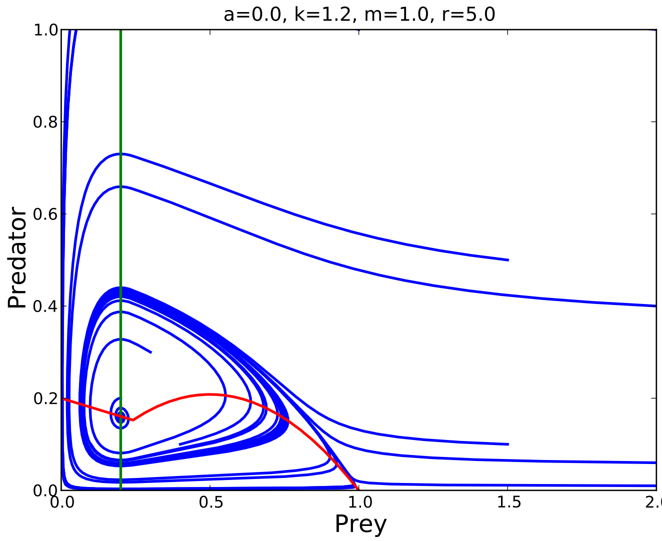
Answer:



There is one saddle-point and one stable focus. There are no limit cycles.

(d) $a = 0., k = 1.2, m = 1, r = 5$

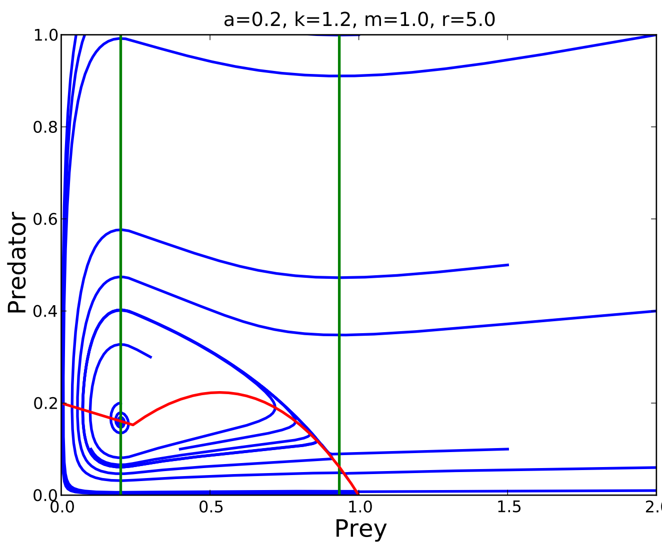
Answer:



There is one saddle point on the boundary, and one stable focus. There are two limit cycles, one stable and one unstable. Depending on initial conditions, almost all solutions approach one of these two stable attractors.

(e) $a = 0.24, k = 1.2, m = 1, r = 5$

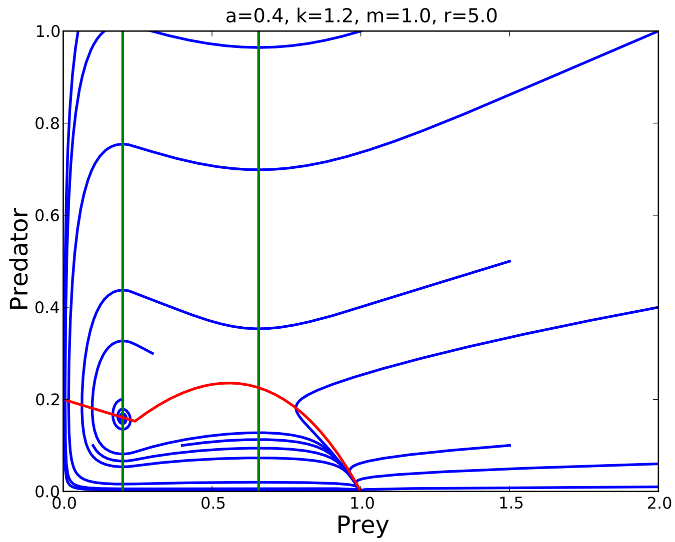
Answer:



There is one stable node (predators extinct), one saddle point, and one stable focus. there are two limit cycles, one stable, one unstable. Depending on the initial conditions, the solution may either oscillate forever, or the predator may go extinct and the prey reaches carrying capacity.

(f) $a = 0.4, k = 1.2, m = 1, r = 5$

Answer:



There is one stable focus, one interior saddle point, one unstable limit cycle, and one attracting node on the boundary, corresponding to predator extinction.

Identify all stationary solutions, and their local stability. Also identify any limit-cycle solutions. (This is all best-done with a numerical ODE solver and some well-chosen initial conditions).