Homework 10, Math 451, Spring 2013

due Monday, April 22th

This homework covers material on ordinary differential equations and optimization.

1. Use Taylor series to determine the local order of precision of the implicit trapezoidal method

$$x(t+h) = \frac{h}{2}\left[f(x(t+h)) + f(x(t))\right] + x(t).$$

to the autonomous differential equation

$$\frac{dx}{dt} = f(x)$$

2. Use the Golden Ratio method to calculate

$$\operatorname*{argmin}_{x\in[-1,1]} \sin(50x)/10 + x^2.$$

3. Use line search starting from initial point $p = (1, 0)$ in the direction of $(-1, -1)$ and with initial step size $h = .1$ to find the first minimum of

$$f(x, y) = \frac{x^3 + 7xy}{1 + x^4 + y^4}.$$

along the parametric line $(x, y) = (2 - s, 1 - s)$ to 4 significant digits.

4. In this problem, we will explore the use of simulated annealing to find the global minimum of $E(x) = x^4 - 10x^2 + 10x$.

   (a) Use the derivative of $E(x)$, root-finding methods, and the second derivative to find the two local minima. Also, determine which is the global minimum.

   (b) Implement simulated annealing to approximate the global minimum. Let your initial state $x_0 = (2 * \text{rand}() - 1) * 4$. Generate neighboring points using $y = x + \text{randn}() * 0.5$. Set your initial temperature to $T_0 = 10$, and use a cooling schedule where at each time step, the temperature decreases by 1 percent. Stop the simulation when $T < 10^{-2}$.

   Run this 200 times, and determine how often simulated annealing successfully finds the global minimum.

   (c) Rerun the simulated annealing procedures, but this time, use $y = x + \text{randn}() * 0.1$.

   How often simulated annealing successfully finds the global minimum in this case?