# Homework 9, Math 451, Spring 2013

due Wednesday, April 10th

This homework covers material on eigenvalue methods from Chapter 11 of Mathews and Fink.

1. Complete Section 11.1, #1, a-c
2. For each of the matrices $A$ in the previous problem, construct the eigenvalue decomposition $V\Lambda V^{-1} = A$.
3. For each of the matrices $A$ in problem 1, determine the spectral radius of the matrix $||A||_2 = \max_i |\lambda_i|$ and the spectral gap $|\lambda_1 - \lambda_2| / \max\{|\lambda_1|, |\lambda_2|\}$.
4. Let
$$M = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}. \tag{1}$$
   (a) Find the dominant eigenvector in normalized form in Matlab using the power method. Stop the process when the 2-norm between two consecutive iterations is less than 0.002. How many iterations did this take?
   (b) Plot the 2-norm of the residual as a function of the number of iterations on a semilogy plot. What does your plot suggest about the convergence rate of the power method?
   (c) Find the associated eigenvalue by hand using matrix multiplication.
5. Provide a succinct description of what the following piece of Matlab code does.

```
function [v,lambda]=a_function(A,tolerance)
n = length(A);
v = randn(n,1);
v = (v / norm(v,2));
v_old = -v;
while ( norm(v-v_old,2) > tolerance )
v_old = v;
try
v = back_sub(pelim( A, v));
catch err
v(1) = 0/0;
end
if sum(isnan(v))>0
v = v_old;
break;
end
v = (v / norm(v,2));
v = v*sign(v(1));
end
lambda = v'*A*v;
return;
```