This homework covers material on eigenvalue methods from Chapter 11 of Mathews and Fink.

1. Complete Section 11.1, #1, a-c

2. For each of the matrices $A$ in the previous problem, construct the eigenvalue decomposition $V \Lambda V^{-1} = A$.

3. For each of the matrices $A$ in problem 1, determine the spectral radius of the matrix $\|A\|_2 = \max_i |\lambda_i|$ and the spectral gap $|\lambda_1 - \lambda_2|/\max\{|\lambda_1|, |\lambda_2|\}$.

4. Let

$$M = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}.$$  

(a) Find the dominant eigenvector in normalized form in Matlab using the power method. Stop the process when the 2-norm between two consecutive iterations is less than 0.002. How many iterations did this take?

(b) Plot the 2-norm of the residual as a function of the number of iterations on a semilogy plot. What does your plot suggest about the convergence rate of the power method?

(c) Find the associated eigenvalue by hand using matrix multiplication.

5. Provide a succinct description of what the following piece of Matlab code does.

```matlab
function [v,lambda]=a_function(A,tolerance)
    n = length(A);
    v = randn(n,1);
    v = (v / norm(v,2));
    v_old = -v;
    while ( norm(v-v_old,2) > tolerance )
        v_old = v;
        try
            v = back_sub(pelim( A, v));
        catch err
            v(1) = 0/0;
        end
        if sum(isnan(v))>0
            v = v_old;
            break;
        end
        v = (v / norm(v,2));
        v = v*sign(v(1));
    end
    lambda = v'*A*v;
    return;
```