Homework 8, Math 451, Spring 2013

due Monday, April 1st

This homework covers material on linear systems, linear least squares methods and curve-fitting of Mathews and Fink, and completes quadrature.

1. Non-Newton-Cotes quadrature
   (a) Use a linear substitution to transform the integral $\int_0^1 \sqrt{1-x^2}dx$ into an integral over the interval $[-1,1]$.
   (b) Apply the Matlab code from class for the Gauss--Legendre rule to obtain an estimate of this integral using 5 points.
   (c) Apply the Matlab code from class for the Clenshaw--Curtis rule to obtain an estimate of this integral using 5 points.
   (d) These methods do not do well for approximating the actual value. Why? (give 2 reasons)
2. Find all solutions of the following equation. Specify the rank of the matrix and the dimension of the column nullspace and the row nullspace of the matrix.

$$\begin{bmatrix} -4 & -2 & 0 & 0 & 0 \\ 1 & 0 & 5 & 0 & -2 \\ -3 & -1 & -2 & 1 & 2 \\ -1 & -1 & 2 & -1 & -2 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

3. For each of the following, determine if the equation is solvable. If it is solvable, give all solutions. If it is not solvable, give the normal equations for the corresponding least-squares problem, and all of the solutions to the normal equations. Freestyle: Use Matlab for all parts, but show each step and its result.

(a)

$$\begin{bmatrix} 4 & -2 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} x = \begin{bmatrix} -0.84147 \\ -0.47943 \\ 0.00000 \\ 0.47943 \\ 0.84147 \end{bmatrix}$$

(b)

$$\begin{bmatrix} -0 & -1 & -2 & -0 \\ 1 & -0 & -0 & 2 \\ 1 & -1 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 2 & -2 & 1 \end{bmatrix} x = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0.1 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} x = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

4. In 1950, the English fluid dynamicist G. I. Taylor used publicly available movie footage and a gorgeous piece of applied mathematics to estimate the energy of Trinity's ball of fire as $17-24$ kilotons of TNT. At the time, this was top-secret information. A key component of the analysis was fitting a curve through the following data:

t (milliseconds)	Explosion Radius R (meters)
0.24	19.9
0.52	28.8
1.93	48.7
3.26	59.0
15.0	106.5

(a) Construct a new table, containing $\log_{10} t$ and $\log_{10} R$.

(b) Assume that data in your new table are close to satisfying an equation of the form

$$m \log_{10} t + b = \log_{10} R.$$

Construct the normal equations for this system by hand and calculator.

(c) Solve the normal equations using LU factorization in Matlab, showing the both the L and U factors and your solution steps.

(d) Plot the transformed data points and the line you estimated on one figure.

(e) Calculate the residual r and the R^2 statistic. Plot the residual as a function of the time t without a line connecting the points. Include the x-axis in your plot for reference.

5. You wake up on an alien world, with no idea how you arrived there. You set out to learn about your world. You happen to have a water bottle that has a mass of of 1 kg. You throw this into the air, and observe the table of heights over time below.

Time t (seconds)	Height h (meters)
0.15	3.7
0.30	5.1
0.46	6.2
0.61	7.1
0.77	7.6
0.92	8.0

(a) Construct the matrix A to be used in your least-squares fit.

(b) Use a computer to find the QR decomposition of A. Include your answers to atleast 3 significant digits.

(c) Use the QR decomposition to approximate the gravitational acceleration on your world based on the equation $h(t) = gt^2/2 + v_0 t + h_0$

6. Carefully read through and run the following piece of code, using Matlab's help for commands you do not recognize.

```
clear all;
max_n = 1024;
dil = 1.3;
largest_exponent=floor(log(max_n)/log(dil));
n_range = floor(dil.^(largest_exponent:-1:1));
n_range = unique(n_range);

for k=1:length(n_range)
  n = n_range(k);
  A=randn(n);
  b=randn(n,1);
  tic;
    x = A\b;
  t_full(k) = toc;

  tic;
    [L,U] = lu(A);
  t_factor(k) = toc;
    x = U\(L\b);
  t_total(k) = toc;
  t_subs(k) = t_total(k) - t_factor(k);
end
n = n_range;
loglog(n,t_full,'bo-', n,t_subs,'gx-',n,t_total,'r+-',  'linewidth',2);
```

(a) Add appropriate labels, legends, and titles to the resulting plot, and tell me what the plot means.

(b) Use an approach similar to that of G. I. Taylor's problem to fit a curve to the green line, using only data points with x values greater than 100. (Warning: For this problem, Octave will give the wrong results. Please use Matlab.)