Homework 6, Math 451, Spring 2013

due Wednesday, March 13

This home covers material from the end of chapter 4 of Mathews and Fink. You may find the Matlab functions ‘‘interp1’’, ‘‘poly’’, and ‘‘polyval’’ useful when answering the problems.

1. Let $g(x) = \cos(x^2)$. Make a Matlab figure with two subplots in one row. In the first subplot, plot $g(x)$ over the domain $[0, 4]$ using 1000 sample points. On the same subplot, plot the nearest-neighbor interpolant of $g(x)$ using 7 equally spaced data points (including the end points). In the second subplot, again plot $g(x)$ over the domain $[0, 4]$ using 1000 sample points. On the same subplot, plot the linear interpolant of $g(x)$ using 7 equally spaced data points (including the end points). You can use ‘‘interp1’’ for this problem.

2. The Gamma function
$$\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx.$$
For non-negative integers n, $\Gamma(n) = (n-1)!$.
   (a) Determine $\Gamma(n)$ for $n = 1, 2, 3, 4$.
   (b) Use the Lagrange interpolation formulas to find an interpolating polynomial through these 4 points.
   (c) Use this polynomial to estimate $\Gamma(2.5)$.

3. Page 220 of Matthews and Fink, # 2. a and c. Skip part b. For (a), you can use any method to calculate the polynomial.

4. As discussed in class, the Chebyshev polynomial of the first kind of order n is represented by $T_n(x)$. Expand the formula $T_n(x) = \cos(n \arccos(x))$ to calculate $T_2(x)$. Show your work. You should make use of trigonometric identities in your answer.

5. Use the table from your book to determine $T_6(x)$. Use this polynomial to determine expand $\cos(6x)$ in powers of $\cos(x)$.

6. Determine the locations of the zeros of $T_5(x)$ to 3 significant digits.

7. Consider the function $f(x) = |x|$. Make a figure with 4 subplots (2x2) as follows. All subplots should have domains of $[-1, 1]$. In the first subplot, make a plot of $|x|$ as a dotted line, and the 12 node polynomial interpolant of $f(x)$ with equally-spaced nodes as a solid line. In the second subplot, plot the error between $f(x)$ and it's interpolant. Use enough samples so that both curves appear smooth. In the third subplot, make a plot of $|x|$ as a dotted line, and the 12 node polynomial interpolant of $f(x)$ with Chebyshev nodes as a solid line. In the fourth subplot, plot the error between $f(x)$ and this interpolant. Observe how the error patterns differ.