Homework 3, Math 451, Spring 2013

due Wednesday, February 6th

This home covers material from Sections 2.4,2.5,3.1,3.2 of Mathews and Fink.

- Book problems
  - Section 3.1, # 1
  - Section 3.2, # 2
  - Section 3.2, # 10
- Non-book problems
  1. Secant method:
     (a) Find the simplified version of the secant-method's fixed-point formula $p_{t+1} = g(p_t, p_{t-1})$ for calculating $\sqrt[3]{a}$.
     (b) Apply your method to approximate $\sqrt[3]{7}$, with $p_0 = 1$, $p_1 = 2$, and calculate p_2, p_3, p_4.
  2. Order of convergence
     (a) Determine the order of convergence of the sequence $p_n = 1 + 1/n^k$ to $p^* = 1$
     (b) Derive the Newton--Raphson formula to solve $f(x) = e^{rx} - 1 - x$, and determine the order of convergence when $r = 1$ to $x^* = 0$.
  3. Muller's method: A function $f(x)$ passes through 3 points, $(-1, 16), (1, 6), (2, -8)$. Use the formula's for Muller's method to find the equation $y = a + bx + cx^2$ for a parabola through the points, and use it to approximate a solution of $f(x) = 0$.
  4. Define w_n as the 3rd smallest positive solution of
     $$\frac{x}{n} - \frac{11}{10} = \sin(x)$$
     , if it exists, or 0, if the 3rd smallest positive solution does not exist. Find
     $$X = \sum_{n=1}^{50} w_n.$$
     Include your Matlab (or other) code as an appendix to this assignment.
  5. Horner's method: Rewrite the first 5 terms of the McLaurin series of e^x using Horner's method to minimize the number of multiplications.