Name:______________________________

Instructions: Clearly answer each of the questions below. Remember to check the back side – if blank, you can use it for scrap work. Use full sentences and proper grammar. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers when appropriate.

1. Given the functions f and r with plots shown below

4				×	
3	×	×			
2					
1					
0			×		×
r	0	1	2	3	4

4	×				
3		×			
2				×	
1			×		
0					×
f	0	1	2	3	4

   (a) Is f as injective, surjective, bijective, or none of these. ______________

   Answer: *f is a bijection*

   (b) Is r as injective, surjective, bijective, or none of these. ______________

   Answer: *r is a none of these*

   (c) Find 2-row representations of f and r.

   Answer:

$$f := \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 & 0 \end{pmatrix} \quad \textit{and} \quad r := \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 3 & 3 & 0 & 4 & 0 \end{pmatrix}$$

   (d) Calculate the function $f^{-1}@r$, and give your answer in two-row form.

   Answer:

$$f^{-1} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 & 0 \end{pmatrix} \quad \textit{and} \quad f^{-1}@r = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 4 & 0 & 4 \end{pmatrix}$$

2. What condition is required of two functions f and g, for their composition $f@g$ to exist?

   Answer: *The composition $f@g$ exists if and only if the image of g is a subset of the domain of f.*

3. What is a binary relation?

   Answer: *A binary relation is a function with a domain that is the Cartesian product of two sets, and a target that is true or false. Our textbook defines a binary relation as a subset of a Cartesian product. These two definitions can be rectified. Given sets A and B, if we have a binary relation defined as a function $R : A \times B \to \{true, false\}$, then then the relation set is $\{(a, b) \in A \times B : R(a, b) = true\}$. If we have a relation set $S \subset A \times B$, then we can define a function $R(a, b)$ such that $R(a, b) = true$ if $(a, b) \in S$ or $R(a, b) = false$ if $(a, b) \notin S$. So the two definitions are equivalent.*

4. What are the differences between a partial ordering, a strict partial ordering, and a total ordering?

   Answer: *A strict partial ordering is a relation that is antisymmetric and transitive. A regular partial ordering is also transitive, but weakly antisymmetric and reflexive, rather than antisymmetric. A total ordering is a complete partial ordering.*

5. For each binary relation below on $\mathbb{N}$, cross out the properties that do not apply.

(a) $a < b$: reflexive, symmetric, weakly antisymmetric, antisymmetric, transitive, complete.

Answer: *antisymmetric and transitive*

(b) $c \geq d$: reflexive, symmetric, weakly antisymmetric, antisymmetric, transitive, complete.

Answer: *weakly antisymmetric, reflexive, transitive, and complete*

(c) $x|y$: reflexive, symmetric, weakly antisymmetric, antisymmetric, transitive, complete.

Answer: *weakly antisymmetric, reflexive, and transitive*

(d) $u \equiv_6 v$: reflexive, symmetric, weakly antisymmetric, antisymmetric, transitive, complete.

Answer: *symmetric, reflexive, and transitive*

6. Rank the elements of the set of vectors $\{(1,1),(1,2),(2,1),(2,2)\}$ using the relation $(x,y) \prec (u,v)$ if and only if $y < v$ or $y = v$ and $x \leq u$. What type of ordering is $\prec$ on this set?

Answer: $(1,1) \prec (2,1) \prec (1,2) \prec (2,2)$. *This is a total ordering.*

7. Define a new binary relation $x \bowtie y$ on $\{2,3,4,6\}$ if and only if x and y are *not* relatively prime.

(a) Draw an adjacency table for $x \bowtie y$.

(b) Prove that $x \bowtie y$ is *not* an equivalence relation.

Answer:

	$x \bowtie y$	*2* (y)	*3* (y)	*4* (y)	*6* (y)
	2	*X*		*X*	*X*
x	*3*		*X*		*X*
	4	*X*		*X*	*X*
	6	*X*	*X*	*X*	*X*

The binary relation $x \bowtie y$ *is not an equivalence relation. While it is both reflexive and symmetric, it is not transitive. While* $2 \bowtie 6$ *and* $6 \bowtie 3$*, this does NOT imply* $2 \bowtie 3$*. Similarly,* $3 \bowtie 6$ *and* $6 \bowtie 4$*, but not* $3 \bowtie 4$*.*