Math 311, Quiz 6

Name:_____

Instructions: Clearly answer each of the questions below. Remember to check the back side – if blank, you can use it for scrap work. Use full sentences and proper grammar. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. Suppose $G = (S, *)$ is a group. Name the group property described by each of the logic sentences below.

   (a) $\forall f \in S \, \forall g \in S \, \forall h \in S \, (f * (g * h) = (f * g) * h)$ Answer: *The operation is associativity*

   (b) $\exists e \in S \, \forall g \in S (g * e = e * g = g)$ Answer: *There is an identity element*

   (c) $\forall g \in S \, \exists j \in S (g * j = j * g = e)$ Answer: *All elements have inverses*

   (d) $\forall f \in S \forall g \in S (f * g \in S)$ Answer: *The set is closed under the operation*

2. What additional property must a group have to be an Abelian group?

   Answer: *A group is an Abelian group if and only if the binary operation is commutative over the set.*

   $$\forall g \in G \, \forall h \in G \, (g * h = h * g)$$

3. Consider the group $H = (\mathbb{Z}_9^\times, \times)$.

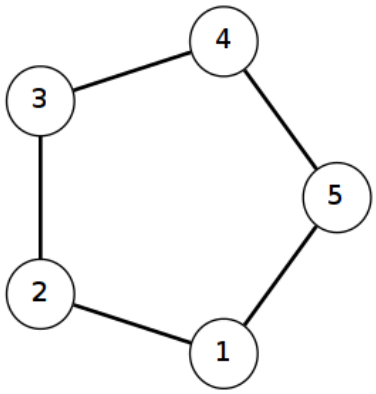
   (a) How many elements are in the group? Answer: $|\mathbb{Z}_9^\times| = \phi(9) = 3^2 - 3 = 6$

   (b) Construct a multiplication table for the group.

   Answer:

ab	b					
	$[1]_9$	$[2]_9$	$[4]_9$	$[5]_9$	$[7]_9$	$[8]_9$
$[1]_9$	$[1]_9$	$[2]_9$	$[4]_9$	$[5]_9$	$[7]_9$	$[8]_9$
$[2]_9$	$[2]_9$	$[4]_9$	$[8]_9$	$[1]_9$	$[5]_9$	$[7]_9$
a $[4]_9$	$[4]_9$	$[8]_9$	$[7]_9$	$[2]_9$	$[1]_9$	$[5]_9$
$[5]_9$	$[5]_9$	$[1]_9$	$[2]_9$	$[7]_9$	$[8]_9$	$[4]_9$
$[7]_9$	$[7]_9$	$[5]_9$	$[1]_9$	$[8]_9$	$[4]_9$	$[2]_9$
$[8]_9$	$[8]_9$	$[7]_9$	$[5]_9$	$[4]_9$	$[2]_9$	$[1]_9$

4. Considering the pentagon below, determine which permutations represent symmetry transformations in **flatland** (2 dimensions), **sphereland** (3 dimensions), or are **not a symmetry** transformation of the labelled pentagon. Note that everything that is a symmetry in 2 dimensions is also a symmetry in 3 dimensions. Give the most useful answer.



(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ Answer:*flatland*

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 3 & 5 \end{pmatrix}$ Answer:*not a symmetry*

(c) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 4 & 5 & 1 \end{pmatrix}$ Answer:*not a symmetry*

(d) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \end{pmatrix}$ Answer:*flatland*

(e) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$ Answer:*sphereland*

(f) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 1 & 2 \end{pmatrix}$ Answer:*flatland*