Math 311, Quiz 6

Name:__________________________________

Instructions: Clearly answer each of the questions below. Remember to check the back side – if blank, you can use it for scrap work. Use full sentences and proper grammar. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers when appropriate.

1. Suppose $G = (S, \star)$ is a group. Name the group property described by each of the logic sentences below.

   (a) $\forall f \in S\, \forall g \in S\, \forall h \in S\, (f \star (g \star h) = (f \star g) \star h)$ Answer: *The operation is associativity*

   (b) $\forall f \in S \forall g \in S (f \star g \in S)$ Answer: *The set is closed under the operation*

   (c) $\exists e \in S\, \forall g \in S (g \star e = e \star g = g)$ Answer: *There is an identity element*

   (d) $\forall g \in S\, \exists j \in S (g \star j = j \star g = e)$ Answer: *All elements have inverses*

2. Let $G = (\mathbb{N}, \bowtie)$, where we define $g \bowtie h = \min(g, h)$. Determine which, if any, of the properties (a)-(d) in problem 1 does G satisfy. For those properties that are not satisfied, give a counter-example.

   Answer: *The operation $g \bowtie h$ on the natural numbers is associative because $\min(\min(x, y), z) = \min(x, \min(y, z))$, and it is closed because $x, y \in \mathbb{N}$ implies $\min(x, y) \in \mathbb{N}$, but there is no identity, and there are no inverses. For example, $\min(4, 6) = 4$, so we might consider 6 for the identity, but $\min(7, 6) = 6$, not 7, so 6 is not an identity element. Also, there is no integer y, such that $\min(5, y) = 6$, so 5 would not have an inverse if 6 was the identity. In some sense, ∞ would be the best choice for an identity element, but ∞ is not a natural number, and we still wouldn't have inverses because $\min(5, y) \neq \infty$ for any y.*

3. Consider the group $G = (\mathbb{Z}_9^\times, \times)$ of invertible congruence classes modulo 9 under multiplication.

   (a) How many elements are in the group? Answer: $|\mathbb{Z}_9^\times| = \phi(9) = 3^2 - 3 = 6$

   (b) List all the elements of $\mathbb{Z}_9^\times$.

   Answer:

   $$\{[1]_9, [2]_9, [4]_9, [5]_9, [7]_9, [8]_9\}$$

   (c) Construct a multiplication table for the group.

   Answer:

	$[1]_9$	$[2]_9$	$[4]_9$	$[5]_9$	$[7]_9$	$[8]_9$
$[1]_9$	$[1]_9$	$[2]_9$	$[4]_9$	$[5]_9$	$[7]_9$	$[8]_9$
$[2]_9$	$[2]_9$	$[4]_9$	$[8]_9$	$[1]_9$	$[5]_9$	$[7]_9$
$[4]_9$	$[4]_9$	$[8]_9$	$[7]_9$	$[2]_9$	$[1]_9$	$[5]_9$
$[5]_9$	$[5]_9$	$[1]_9$	$[2]_9$	$[7]_9$	$[8]_9$	$[4]_9$
$[7]_9$	$[7]_9$	$[5]_9$	$[1]_9$	$[8]_9$	$[4]_9$	$[2]_9$
$[8]_9$	$[8]_9$	$[7]_9$	$[5]_9$	$[4]_9$	$[2]_9$	$[1]_9$