Math 311, Quiz 5

Name:____

Instructions: Clearly answer each of the questions below. Remember to check the back side – if blank, you can use it for scrap work. Use full sentences and proper grammar. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers when appropriate.

1. Translate the following sentence into its symbolic predicate logic form: "For every integer x, there exists a natural number y such that $x^2 = y$."

   Answer: $\forall x \in \mathbb{Z} \exists y \in \mathbb{N}(x^2 = y)$

2. Given the sets $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 5, 6\}$, and $C = \{1, 3, 5, 7\}$, find $(A \backslash C) \cap B$.

   Answer: $A \backslash C = \{2, 4\}$, *so* $(A \backslash C) \cap B = \{2\}$.

3. For each of the following, is the given set a partition of the set $\{1, 2, 4, 8, 9\}$? If not, why?

   (a) $\{\{4, 2\}, \{8, 1\}, \{9, 4\}\}$. Answer: *No, 4 appears in two elements, so the elements are not disjoint.*

   (b) $\{\{1, 4, 8\}, \{2, 9\}\}$. Answer: *Yes, this is a partition. The elements don't overlap, and their union gives us back* $\{1, 2, 4, 8, 9\}$.

   (c) $\{\{4, 8\}, \{2, 9\}\}$. Answer: *No, the union of the elements does not include 1, but 1 is an element of the original set.*

4. If $Y = \{a, b\}$, and $Z = \{7, 5, a\}$, list all the elements of $Z \times Y$.

   Answer: $\{(7, a), (5, a), (a, a), (7, b), (5, b), (a, b)\}$

5. Fill in the blanks. A function is Answer: *injective* if no two arguments from the domain have the same image. A function is Answer: *surjective* if every element of its codomain is the image of atleast one argument from the domain. If the function satisfies both of these conditions, we call it Answer:*bijective*.

6. Which among the following sets are equal to one another?

$$X = \{x \in \mathbb{Z} : x^3 = x\}, \quad Y = \{x \in \mathbb{Z} : x^2 = x\},$$
$$Z = \{x \in \mathbb{Z} : x^2 \leq 2\}, \quad W = \{0, 1, -1\}, \quad V = \{1, 0\}.$$

   Answer: *Note that* $x^3 = x$ *means* $0 = x - x^3 = x(1 - x)(1 + x)$ *and* $x = x^2$ *means* $x - x^2 = x(1 - x) = 0$. *And so* $W = X = Z$ *and* $V = Y$. *(This was a homework problem.)*