

Name: _____

Instructions: Clearly answer each of the questions below. Remember to check the back side – if blank, you can use it for scrap work. Use full sentences and proper grammar. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. Given that sets $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 5, 6\}$, and $C = \{1, 3, 5, 7\}$, find $(A \cap B) \setminus C$.

Answer: The intersection of A and B, excluding the elements of C is {2}

2. Give two reasons why $\{\{1, 2\}, \{1, 4\}, \{5\}\}$ is not a partition of the set $\{1, 2, 3, 4, 5\}$.

Answer: A partition is made up of non-overlapping subsets that cover the whole of the original set. In this case, 1 is in both of the first two, so two of the subsets overlap. Also, 3 is not in any one of the pieces, so the subsets do not completely cover the original set.

3. If $|A| = 10$, $|B| = 8$, and $|A \cap B| = 2$, then what is $|A \cup B|$?

Answer: $|A \cup B| = |A| + |B| - |A \cap B| = 10 + 8 - 2 = 16$

4. If $Y = \{1, 2\}$ and $Z = \{3, 2, 1\}$, list all the elements of the set $Y \times Z$.

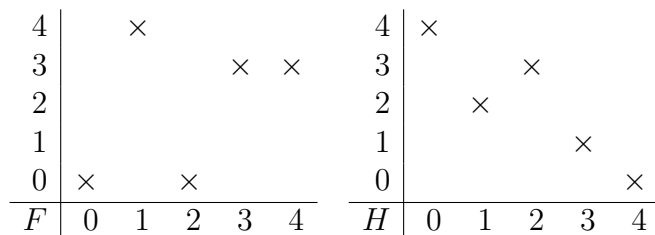
Answer: $Y \times Z = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$. Note that the pairs of elements are ordered. (1, 3) is in the Cartesian product, but (3, 1) is not. We have to use parenthesis instead of braces to make this ordering clear. Braces would indicate sets, which are not ordered.

5. Which among the following sets are equal to one another?

$$X = \{x \in \mathbb{Z} : x^3 = x\}, \quad Y = \{x \in \mathbb{Z} : x^2 = x\}, \\ Z = \{x \in \mathbb{Z} : x^2 \leq 2\}, \quad W = \{0, 1, -1\}, \quad V = \{1, 0\}.$$

Answer: Note that $x^3 = x$ means $0 = x - x^3 = x(1 - x)(1 + x)$ and $x = x^2$ means $x - x^2 = x(1 - x)$. And so $W = X = Z$ and $V = Y$. (This was a homework problem.)

6. Given the functions F and H with plots shown below



- (a) Is F as injective, surjective, bijective, or none of these.

Answer: F is a none of these

- (b) Is H as injective, surjective, bijective, or none of these.

Answer: H is a bijection

- (c) Find 2-row representations of functions F and H .

Answer:

$$F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 4 & 0 & 3 & 3 \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 & 0 \end{pmatrix}$$

- (d) Solve $H@X = F$ for the unknown function X , if possible, by calculating $H^{-1}@F$. *Answer:*

$$H^{-1} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 & 0 \end{pmatrix}$$

so

$$X = H^{-1}@F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 4 & 0 & 4 & 2 & 2 \end{pmatrix}$$