

Name: _____

Instructions: Clearly answer each of the questions below. Remember to check the back side – if blank, you can use it for scrap work. Use full sentences and proper grammar. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers when appropriate.

1. List the truth-table definitions for the logical operations $p \wedge q$ and $\neg p$.

Answer:

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2. Identify the following logic identities.

(a) $(p \vee (q \wedge p)) \leftrightarrow p$

Answer: *Absorption law*

(b) $(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$

Answer: *Associative Law of Conjunction*

(c) $(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$

Answer: *A Distributive Law*

(d) $(\neg p \wedge \neg q) \leftrightarrow \neg(p \vee q)$

Answer: *One of De Morgan's Laws*

3. Construct a truth-table for the following statement of propositional logic and determine if it is a tautology, contingency, or contradiction.

$$((y \wedge \neg w) \vee \neg x) \wedge (x \rightarrow w)$$

Answer:

w	x	y	$\neg x$	$\neg w$	$y \wedge \neg w$	$(y \wedge \neg w) \vee \neg x$	$x \rightarrow w$	$((y \wedge \neg w) \vee \neg x) \wedge (x \rightarrow w)$
T	T	T	F	F	F	F	T	F
T	T	F	F	F	F	F	T	F
T	F	T	T	F	F	T	T	T
T	F	F	T	F	F	T	T	T
F	T	T	F	T	T	T	F	F
F	T	F	F	T	F	F	F	F
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	T	T	T

4. Use the definitions to express the logical proposition $(r \wedge (q \vee \neg p)) \rightarrow s$ as a plain English sentence.

$p =$ Ms. Who knows when

$r =$ Meg can save Charles Wallace

$q =$ Ms. Whatsit knows where

$s =$ Ms. Which knows why

Answer: If Meg can save Charles and Ms. Whatsit knows where or Ms. Who doesn't know when then Ms. Which knows why.

5. Prove the tautology $p \rightarrow (\neg p \rightarrow q)$ by deduction.

$p \rightarrow (\neg p \rightarrow q)$ *Given*

$p \rightarrow (\neg\neg p \vee q)$ *Definition of Implication*

$p \rightarrow (p \vee q)$ *Double negative*

Answer: $\neg p \vee (p \vee q)$ *Definition of Implication*

$(\neg p \vee p) \vee q$ *Associativity of Disjunction*

$T \vee q$ *Law of the Excluded Middle*

T *Part of the Definition of Truth*