

Name: \_\_\_\_\_

Instructions: Clearly answer each of the questions below. Remember to check the back side – if blank, you can use it for scrap work. Use full sentences and proper grammar. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. List the truth-table definitions for the logical operations  $\neg p$  and  $p \vee q$ .

Answer:

$p$	$\neg p$
$T$	$F$
$F$	$T$

$p$	$q$	$p \vee q$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$F$

2. Identify the following logic identities.

(a)  $(p \vee \neg p) \leftrightarrow T$

Answer: *Law of the Excluded Middle*

(b)  $(p \vee r) \leftrightarrow (r \vee p)$

Answer: *Commutative Law of Disjunction*

(c)  $(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \vee r))$

Answer: *A Distributive Law*

(d)  $(\neg p \vee \neg q) \leftrightarrow \neg(p \wedge q)$

Answer: *One of De Morgan's Laws*

3. Construct a truth-table for the following statement of propositional logic and determine if it is a tautology, contingency, or contradiction.

$$((y \wedge \neg w) \vee \neg x) \wedge (x \rightarrow w)$$

Answer:

$w$	$x$	$y$	$\neg x$	$\neg w$	$y \wedge \neg w$	$(y \wedge \neg w) \vee \neg x$	$x \rightarrow w$	$((y \wedge \neg w) \vee \neg x) \wedge (x \rightarrow w)$
$T$	$T$	$T$	$F$	$F$	$F$	$F$	$T$	$F$
$T$	$T$	$F$	$F$	$F$	$F$	$F$	$T$	$F$
$T$	$F$	$T$	$T$	$F$	$F$	$T$	$T$	$T$
$T$	$F$	$F$	$T$	$F$	$F$	$T$	$T$	$T$
$F$	$T$	$T$	$F$	$T$	$T$	$T$	$F$	$F$
$F$	$T$	$F$	$F$	$T$	$F$	$F$	$F$	$F$
$F$	$F$	$T$	$T$	$T$	$T$	$T$	$T$	$T$
$F$	$F$	$F$	$T$	$T$	$F$	$T$	$T$	$T$

4. Use the definitions to express the logical proposition  $(q \vee s) \rightarrow (r \wedge p)$  as a plain English sentence.

$p =$  beware of the jabberwocky

$r =$  Euclid was mistaken

$q =$  all mimsy are the borogoves

$s =$  Azathoth sleeps

*Answer: If all mimsy are the borogoves or Azathoth sleeps then Euclid was mistaken and you must beware of the jabberwocky.*

5. Prove the tautology  $p \rightarrow (\neg p \rightarrow q)$  by deduction.

$p \rightarrow (\neg p \rightarrow q)$     *Given*

$p \rightarrow (\neg\neg p \vee q)$     *Definition of Implication*

$p \rightarrow (p \vee q)$     *Double negative*

*Answer:*  $\neg p \vee (p \vee q)$     *Definition of Implication*

$(\neg p \vee p) \vee q$     *Associativity of Disjunction*

$T \vee q$     *Law of the Excluded Middle*

$T$     *Part of the Definition of Truth*