Math 311, Quiz 1

Name:______________________________

Instructions: Clearly answer each of the questions below. Remember to check the back side – if blank, you can use it for scrap work. Use full sentences and proper grammar. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers when appropriate.

1. State the Division theorem.

   Answer:*Given any integer b and any positive integer a, then there is a* **unique** *pair of integers q and r where $0 \leq r < a$ such that $b = aq + r$*

2. Why is the Well-ordering principle needed in the proof of the division theorem?

   Answer:*In the proof of the division theorem, we need to find the smallest possible non-negative remainder when dividing b by a. The well-ordering principle assures us that this smallest-remainder actually exists. Remember, not all sets have a smallest element.*

3. Suppose we wish to prove that for all positive integers n,

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

   (a) Show the base case is true.
   Answer:*If $n = 1$, then $\sum_{k=1}^{n} k^2 = 1^2 = 1$ and $n(n+1)(2n+1)/6 = 1(2)(3)/6 = 1$. So*

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

   when $n = 1$.

   (b) What induction hypothesis would you need to show is true to complete the proof? (do not prove)
   Answer: *The induction hypothesis states that if*

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6},$$

   then

$$\sum_{k=1}^{n+1} k^2 = \frac{(n+1)(n+2)(2(n+1)+1)}{6}.$$

4. Finding a counterexample of 3 numbers a, b, and c for which the following conjecture is false: *"If $a|c$ and $b|c$, then $ab|c$."*

   Answer:*There are many possible answers, but one example is $a = 6$, $b = 15$, $c = 30$. $6|30$ and $15|30$, but $6 \times 15 = 90$ and 90 does not evenly divide 30.*

5. Calculate $\gcd(3127, 3233)$.

   Answer:

$$\gcd(3127, 3233) = \gcd(3127, 3127 + 106) = \gcd(3127, 106) =$$
$$\gcd(29(106) + 53, 106) = \gcd(53, 106) = 53.$$