Math 311, Quiz 1

Name:__________________________________

Instructions: Clearly answer each of the questions below. Remember to check the back side – if blank, you can use it for scrap work. Use full sentences and proper grammar. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. What is the Well-ordering principle?

   Answer: *The Well-ordering principle says every set of positive integers has a smallest element.*

2. State the Division theorem.

   Answer: *Given any integer b and any positive integer a, then there is a* **unique** *pair of integers q and r where $0 \leq r < a$ and $b = aq + r$*

3. Supposed we wished to prove

   $$9|4(8^n) + 5(17^n)$$

   for all $n \in \mathbb{N}$ using induction.

   (a) What induction hypothesis would you want to show holds? (do not prove)

   Answer: *If $9|4 \times 8^n + 5 \times 17^n$, then it must also be true that $9|4 \times 8^{n+1} + 5 \times 17^{n+1}$.*

   (b) What additional information would you need complete the proof?

   Answer: *We would also need to establish the base case that $9|4 \times 8^n + 5 \times 17^n$ when $n = 0$.*

4. Finding 3 numbers a, b, and c for which the following conjecture is **false**: *"If $a|bc$, then $a|b$ or $a|c$."*

   Answer: *There are many possible choices. One possibility is $a = 6$, $b = 10$, and $c = 15$. $6|150$ since $6 \times 25 = 150$, but 6 does not evenly divide 10 or 15. The important thing in finding a counter-example is making sure a is composite, and can be factored such that part of a is in b, and the remaining factors of a are all in c.*

5. Calculate $\gcd(713, 589)$.

Answer: *We apply Euclid's algorithm to solve this question.*

$$\begin{aligned} 713\%589 &= 124, \\ 589\%124 &= 93, \\ 124\%93 &= 31, \\ 93\%31 &= 0 \end{aligned}$$

Thus, according to the theorems we proved in class, $\gcd(713, 589) = 31$.