

Name: _____

Math 311w, Section 5, Exam 1

Instructions: Clearly answer each of the questions. Partial credit will be awarded based on the clarity and correctness of your explanation of each solution.

1. (3 pts each) Supply definitions or explanations for each of the following.

(a) $n|a$

(b) $\gcd(a, b)$

(c) a prime number

(d) $x \equiv_n y$ or $x \equiv y \pmod n$

(e) $[x]_n$

(f) $[x]_n^{-1}$

2. (15 pts) Find the greatest common divisor of 546 and 432 and express your answer as a linear combination of 546 and 432.

3. (16 pts) Find all the solutions of each of these two linear recurrence equations:

(a) $3x \equiv_5 4$

(b) $6w \equiv_4 4$

4. (5pts) State the “Division Theorem”.

5. (15 pts) Show that for every positive integer n , $13^n + 5 \times 7^n$ is evenly divisible by 6.

6. (5 pts) State the “Prime Factorization Theorem”, also known as the “Fundamental Theorem of Arithmetic”.
7. (14 pts) Show that there are infinitely many prime numbers.

8. (14 pts) Suppose that for integers a, b , there is an integer c such that b evenly divides $ac - 1$. Show that the greatest common divisor of a and b is 1.