1. Prove that
$$3(14^n) + 10(27^n)$$
is evenly divisible by 13 for all non-negative integer values of n.

2. Prove that
$$17|5(19^n) + 7$$
for all positive integer values of n, or provide a counterexample.

3. Prove that
$$17|4(18^n) + 13$$
for all positive integer values of n, or provide a counterexample.

4. Prove that
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

5. Use deduction to prove $q \to (r \to q)$.

6. Use deduction to prove $((p \wedge q) \to r) \to (p \to r)$.

7. Use deduction to prove $p \wedge (p \to q)$.

8. Prove that $a|b$ is a partial ordering of the positive integers.

9. Show that $a|b$ is not a total ordering of the positive integers.

10. Prove that $a|b$ is a total ordering of the set of positive powers of 2.

11. Suppose G is a group and H is a subgroup of G. Prove that the relation $aRb \iff (\exists k \in H : ak = b$ is an equivalence relation on G.

12. Show that the set of rational numbers is not a group under multiplication.

13. Show that the set of all linear functions $f(x) = ax + b$ where $a \neq 0$ forms a group under function composition with identity function $id(x) = x$.

14. Prove that $\mathbb{Z}_9^\times$ is a cyclic group.

15. Prove that $\mathbb{Z}_8^\times$ is not a cyclic group.